

THE DEMAND FOR HOUSING: APPLICATION
OF A PORTFOLIO-BALANCE MODEL

By

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To my parents

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Abstract of Dissertation Presented to the Graduate
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THE DEMAND FOR HOUSING: APPLICATION OF
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This thesis provides a model of the demand for housing within a portfolio-balance framework. In this framework, the housing choice is jointly determined by the consumption and investment decisions that the household makes in order to maximize utility derived from its returns on assets, subject to the wealth constraint. With some reasonable assumptions about marginal utility of consumption, interest rate on housing mortgage, and the effect of the U.S. income tax, the model predicts that the demand for housing is determined by labor income, wealth, and the price of housing.

The estimation of the model was based on a cross-sectional sample and the use of a heteroscedastic tobit analysis. The sample consists of home owners as 66 percent and non-home owners as 34 percent. Thus, without any

separation of the data, an estimation technique such as the tobit analysis seemed to be appropriate for the treatment of the truncation in the sample. On the other hand, a test for homoscedasticity confirmed the presence of heteroscedasticity and its effects on the results. Therefore, a heteroscedastic tobit analysis was utilized to get consistent and unbiased estimates of the housing demand function.

The thesis argues that the estimation of the income elasticity of the demand for housing based on labor income and wealth will improve and increase the estimate of the elasticity. The standard approach of using total or permanent total income, because of the simultaneous equations bias, underestimates the value of the income elasticity. A comparison between the two methods revealed that the estimate of the income elasticity based on labor income and wealth is higher than that based on total or permanent total income. Thus, although the methods are similar in measuring the income elasticity, their estimates of this elasticity are found to be different.

The results reported in the thesis are likely to be biased due to the lack of data on the housing price index, and hence its omission from the estimates of the housing demand function. The magnitude and direction of the bias depend on the sum of the housing price variations among the cities and regions of the U.S.A. from where the sample was drawn.

CHAPTER I

INTRODUCTION

The importance of the question of income elasticity of the demand for housing lies in the fact that several conclusions in the fields of urban economics, urban planning, economic growth, and economic development have been based on specific magnitudes of the income elasticity of the demand. First, Aaron [3] and Laidler [25] showed that the property tax will be progressive if the income elasticity is greater than unity. Under the special treatment of the U.S. federal income tax laws, therefore, home ownership is stimulated among the high-income households more than among the low-income ones. Contrarily, Edelstein [11] and Lee [28] concluded that the property tax is regressive, on the basis of their estimates that the income elasticity of the demand for housing is less than unity. Second, Mills [34] concluded that the location of dwellings with respect to the central business district depends on the income elasticity of the demand for housing: the higher the income elasticity, the farther the location of the dwelling. Third, Mills' model [34] demonstrated that a high income elasticity of the demand for housing would result in an increase in the total land area of a city, and also an increase in the land value.

Fourth, Reid [43] and Aaron [2] held that a high income elasticity of the demand for housing would increase capital formation in the housing sector. Thus, the growth of the GNP would not accelerate, under the assumption that housing investments are not productive. Furthermore, Aaron showed that such a pattern of growth would result in a rise in the prices of capital-intensive goods relative to the prices of other goods. Fifth, Muth [37] illustrated that the rental value of different housing stocks is directly proportional to their remaining life. Accordingly, he asserted, the factor of proportionality varies directly with income and is larger, the larger the income elasticity of the demand for housing. Finally, Lakshmanan, Chatterjee, and Kroll [26] investigated the demand for housing in relation to the level of economic development. They found that the income elasticity of the demand for housing is low in the early levels of development, increasing during the middle levels, and declining in the high levels of development.

These estimates of the income elasticity of the demand for housing were theoretically justified by different models of the household demand for housing. One of these models was the familiar utility-maximization model where the demand for housing is visualized as the demand for the housing services provided by the housing stock. The relationship between income and expenditures on housing was the main concern of these studies that applied such a model. The other model was the capital-adjustment model where the demand for the housing stock is analyzed within some

frameworks of economic growth such as investment, inflation, and the like. Both models, however, have emphasized one feature of the housing stock: either the consumption-service or asset-acquisition of the housing stock. This suggests that there are still some aspects of the relationship between income and the demand for housing that need to be considered in order to answer completely the question of the magnitude of the income elasticity.

This dissertation argues that the demand for housing can be analyzed within a framework of the portfolio-balance approach where the housing choice is jointly determined by the consumption and investment decisions that the household makes in order to maximize utility derived from its returns on assets, subject to the wealth constraint. In this framework, the investor household has the problem of allocating a given stock of wealth among alternative forms of assets according to the prices of and returns on assets. The model emphasizes the wealth effect on the household trade-off between the consumption of the housing services, as the returns on the housing stock, and the consumption of all other goods and services, including the services of the other assets in the portfolio structure of the household. In addition, the different features of the U.S. income tax have been incorporated into the model to capture their effects on the household choice in the housing market.

One useful aspect of our model, compared with the standard approach of using total or permanent total income

to estimate the income elasticity of the demand for housing, is the treatment of the simultaneous equations bias in the estimation of the income elasticity. Tactically, the standard approach utilizes total or permanent total income of the household as a regressor in the housing demand equation without a distinction between labor income, as exogenously determined, and total income which is determined by the household asset holdings. Thus, the estimates of the income elasticity of the demand for housing are biased and inconsistent due to the correlation between total income and the error term in the housing demand equation. Therefore, to avoid such a simultaneity problem, our model suggests the regression of the quantity demanded for the housing stock on labor income and wealth which are exogenously determined. A comparison between the estimate of the income elasticity of the demand for housing based on our approach with that based on the standard approach will be provided in order to test for the differences between the two approaches.

The second useful aspect of our model is the inclusion of the points which lie off the housing demand curve, where the non-negativity constraint is often binding, in predicting the elasticities of the demand for housing. We will use the tobit analysis to estimate these elasticities from a sample that consists of home owners as 66 percent of the sample and non-home owners as 34 percent. The tobit analysis seems to be the appropriate estimation technique where useful information, such as the probability of holding the

housing stock in the portfolio structure of the household, is utilized in measuring the elasticities of the demand for housing. In addition, we provide a test for homoscedasticity to rationalize the use of a heteroscedastic tobit analysis where the measurements of these elasticities are found to be different.

Therefore, our answer to the question of the income elasticity of the demand for housing is expected to be more useful than the other approaches, in both the way it looks at the determination of the demand for housing and in the way the elasticities of such a demand are predicted.

The organization of this dissertation is the following. Chapter II provides a review of the literature where we pinpoint the need for a portfolio-balance model to explain the demand for housing. In Chapter III we present a model that is built on the interdependency of the consumption and investment decisions of the household in determining its choice in the housing market. Chapter IV illustrates some features of our housing demand function compared with its counterparts in the previous studies on housing. In addition, the chapter includes the specification of our housing demand function and the estimation technique, the tobit analysis, which we use to provide our estimates. Chapter V presents our empirical results, and a comparison between the estimate of the income elasticity of the demand for housing based on our approach and that based on the standard

approach. Chapter VI provides the conclusions of this dissertation. Finally, the appendices show a statistical summary of the data we use to estimate our housing demand equation, and the derivation of the elasticity measurements in the tobit analysis, respectively.

CHAPTER II

THE INCOME ELASTICITY OF THE DEMAND FOR
HOUSING: A REVIEW OF LITERATURE

II.1 Introduction

Although there have been many attempts to estimate the income elasticity of the demand for housing, an accurate estimate is still of concern to many investigators. The estimates of the elasticity range variously between 0.25 and 2.40. These results are not promising because they provide too much latitude for argument and error among policymakers.

Table 1 summarizes the results of 32 studies we surveyed for the period between 1942 and 1979. Column one indicates the author and the year of study, and column two contains his (or her) estimate of the income elasticity of the demand for housing by owners and/or renters. Columns three and four show the variables used to estimate the relationship between housing demand and its determinants. Finally, the fifth column of the table illustrates the source and types of data used in the estimation of this relationship.

The survey of these studies indicates the different methods of analysis, specification of the model, data

TABLE 1

SUMMARY OF THE STUDIES AND THEIR ESTIMATES OF THE INCOME
ELASTICITY OF THE DEMAND FOR HOUSING

(1)	(2)	(3)	(4)	(5)
Author (year)	Income Elasticity of Demand for Housing	Dependent Variable (Housing Demand)	Independent Variables (Income, Price, and DF or Other Variables)	Data Source & Type
Allen[4] (1942)	0 = .33 to .67	Expenditures on housing & utilities	Current income	C, Micro
Winnick[58] (1955)	0 = .5	Per capita house value	Current income	T, Macro
Prais & Houthakker [40](1955)	R = .83	Rent expenditures	Current income	C, Micro
Friend & Kravis[15] (1957)	O & R = .65 to .86	Expenditures on housing	Current income	C, Micro
Maisel & Winnick[32] (1960)	O = .624 R = .557	Expenditures on housing & utili- ties	After-tax income	C, Macro
Muth[35] (1960)	(1) O = 1.0 (2) O = 1.7	House value for a family	Family income, cost construction, & DF	(1) T, Macro (2) C, Macro

Continued

TABLE 1
CONTINUED

(1) Author (year)	(2) Income Elasticity of Demand for Housing	(3) Dependent Variable (Housing Demand)	(4) Independent Variables (Income, Price, and DF or Other Variables)	(5) Data Source & Type
Reid[44] (1962)	0 = 1.5 to 2.05	Average housing expenditures	(1) Average income, price of house, & DF (2) Permanent income, & same variables	C, Macro
Lee[27] (1963)	0 = .89	Cost of house to consumer	Disposable income, & amount of mortgage, & DF	C, Micro
Lee[28] (1964)	(1) 0 = .336 (2) 0 = .809	Desired housing stock per family	(1) Real income per family, relative price of house, & other vari- ables (2) Permanent income, with the same variables	(1) T, Macro (2) T, Macro
Lee[29] (1968)	(1) 0 = .37 to .76 (2) R = .29 to .61	(1) House value (2) Housing Con- sumption	(1) Permanent income, price of house, & DF (2) Permanent income, rent, & DF	(1) C, Micro (2) C, Micro

Continued

TABLE 1
CONTINUED

(1)	(2)	(3)	(4)	(5)
Author (year)	Income Elasticity of Demand for Housing	Dependent Variable (Housing Demand)	Independent Variables (Income, Price, and DF or Other Variables)	Data Source & Type
Winger[57] (1968)	O = 1.05	Property value	Current income, credit conditions, & cost of mortgage	C, Macro
Houthakker & Taylor[18] (1970)	O = 2.4 R = 1.5	Per capita person- al consumption on housing	Total consumption	T, Macro
Houthakker & Taylor[18] (1970)	O = 1.49 R = 0.43	Per capita person- al consumption on housing	Total consumption	C, Micro
DeLeeuw[10] (1971)	(1) O = 1.34 (2) R = 0.81	(1) Median housing expenditures (2) Median rental expenditures	(1) Permanent income, & a price index (2) Median income, rent, & a price index	C, Macro
Maisel et al.[33] (1971)	O = .62	Prospective hous- ing expenditures of a new home buyer	Net effective income, a cost index, & an index of total budget	C, Micro
		Continued		

TABLE 1
CONTINUED

(1)	(2)	(3)	(4)	(5)
Author (year)	Income Elasticity of Demand for Housing	Dependent Variable (Housing Demand)	Independent Variables (Income, Price, and DF or Other Variables)	Data Source & Type
Kain & Quigley[19] (1972)	income coefficient 0 = 0.026	Probability of home ownership	Current income, saving position, and DF	C, Micro
Kartman[21] (1972)	0 = 2.0	Housing stock per capita	Current income, price, population growth, & vacancies	C, Macro
Carlner[8] (1973)	0 = .74 R = .52	House value deflat- ed by housing price index	Different definitions of permanent income, price index of all goods, & DF	C, Micro
Straszheim [52](1973)	0 = .396	Probability of home ownership	Current income, prices, rent, & other variables	C, Micro
Arcelus & Meltzer[6] (1973)	R = .94	Agg. # of houses units demanded	Permanent income, rent, real stock of base money, & real stock of government debt	T, Macro

Continued

TABLE 1
CONTINUED

(1)	(2)	(3)	(4)	(5)
Author (year)	Income Elasticity of Demand for Housing	Dependent Variable (Housing Demand)	Independent Variables (Income, Price, and DF or Other Variables)	Data Source & Type
Birnbaum & Weston[7] (1974)	income coefficient $0 = 0.018$	Probability of home ownership	Current income, non- human wealth, & DF	C, Micro
King[23] (1976)	$0 = .64$ to 2.04	Housing character- istics	Current income, price of each characteristic, & DF	C, Micro
Vaughn[56] (1976)	(1) $O = 1.88$ (2) $R = .32$	(1) Median house value (2) Median gross rent	Permanent income, price of housing services or rent, & DF	C, Macro
Lee & Kong [30](1977)	$O = .31$ to $.70$ $R = .46$ to $.87$	Housing consump- tion	Permanent income, ratio of price of hous- ing to price of non- housing goods, & DF	C, Micro
Roistacher [45](1977)	$O = .34$ $R = .22$	Annual real expen- ditures on housing services	Permanent income, houses price index or rent, & DF	C, Micro

Continued

TABLE 1
CONTINUED

(1) Author (year)	(2) Income Elasticity of Demand for Housing	(3) Dependent Variable (Housing Demand)	(4) Independent Variables (Income, Price, and DF or Other Variables)	(5) Data Source & Type
Polinsky[38] (1977)	R = .46 to .90	Expenditures on housing deflated by a price index	Permanent income, housing price index, & DF	C, Micro
Lakshmanan et al.[26] (1978)	O, R = .25 to 2.33	Per capita housing expenditures	Per capita income	Cross Coun- tries, Macro
Stegman & Sumka[51] (1978)	R = .25 to .40	Annual contract rent	Different definitions of income with DF	C, Micro
Smith & Campbell[50] (1978)	(1) O = .51 (2) O = .65	Housing expendi- tures	Current income with DF	(1) C, Micro (2) C, Macro
Trost & Lee [55](1978)	O = .55 R = .50	Expenditures on housing deflated by a price index	Real permanent income, housing price index, & DF	C, Micro
Continued				

TABLE 1

CONTINUED

(1)	(2)	(3)	(4)	(5)
Author (year)	Income Elasticity of Demand for Housing	Dependent Variable (Housing Demand)	Independent Variables (Income, Price, and DF or Other Variables)	Data Source & Type
Rosen[46] (1979)	0 = .692	Housing expenditures deflated by a housing price index	Permanent income, housing price index, & DF	C, Micro
Rosen[47] (1979)	0 = .35	Value of houses deflated by a housing price index	Permanent income, housing price index, & DF	C, Micro
Polinsky & Ellwood[39] (1979)	(1) 0 = .39 (2) 0 = .57	Expenditures on housing deflated by a price index	Permanent income and a housing price index	(1) C, Micro (2) C, Macro

O = Owners; R = Renters; C = Cross-Sectional data; T = Time-Series data;
 Micro = Data from Individual Observations; Macro = Data from Grouped Observations;
 DF = Demographic Factors; and (1) or (2) indicates a different equation was estimated.

types, estimation techniques, and variables used to estimate the housing demand function. Naturally, these differences lead to disagreement among these studies on the value of the income elasticity of the demand for housing. Nevertheless, efforts have been made to reconcile the estimates of many of these studies by showing the areas of disagreement and the causes of differences. For example, see deLeeuw [10], Polinsky [38], and Vaughn [56].

Summarized below are some of the sources of differences among these studies as well as areas of reconciliation.

1. No single measure of the amount of housing is universally used. Table 1 shows that most studies used household expenditures on housing divided by a housing price index to get a proxy for the housing concept. The others used per capita value of the house, house value, median house value, and rent to overcome the difficulty surrounding the measurement of the quantity demanded for housing. DeLeeuw [10] showed that the use of market value of the home, rather than housing expenditures, as a proxy for the quantity demand for housing, results in an increase in the estimate of the income elasticity. That is, the income elasticity of the demand for housing based on market value is higher than that based on housing expenditures.

2. For the price of housing, Table 1 illustrates that several authors omitted the housing price index from the estimates of the housing demand function, while others used different housing price indices to solve the problem of

specification. According to Polinsky [38], and Polinsky and Ellwood [39], very few of the latter group of researchers correctly specified the housing demand function because they used irrelevant housing price indices. For example, Polinsky [38] showed that the use of metro housing price indices ignores price variations among observations from a given metropolitan area. This causes a downward bias in the estimate of the income elasticity and an upward (toward zero) bias in the estimate of the housing price elasticity.

3. For the measurement of income, Table 1 shows that the earlier studies used current income, per capita income, and disposable income. However, recent studies have emphasized the use of permanent income as the appropriate scale variable. Their rationale is that the household looks beyond the current period's income in making its housing demand decision. With the ability to borrow against future income, the household can spread out housing consumption over a fairly long period of time that is consistent with its permanent income. This indicates that housing expenditures are closely related to permanent income. Therefore, the income elasticity of the demand for housing measured by permanent income is higher than that measured by current

income due to the elimination of the errors-in-variables problem.¹

4. For cross-sectional data, Table 1 indicates that the income elasticity of the demand for housing based on grouped data is higher than that based on ungrouped data. DeLeeuw [10] and Vaughn [56] attributed this difference to the use of mean income in grouped data. That would provide a proxy for permanent income by cancelling out the transitory component of current income through the averaging process.

5. For demographic variables, Table 1 illustrates that several authors made extensive use of these variables, such as family size, education, and age of the household head, as an attempt to control for their effects on housing demand. Other authors did not account for such factors in their estimates of the housing demand function. According to Lee [29], the inclusion of demographic variables would reduce the chances of overestimating the income

¹Generally, Cramer [9] shows that such a measurement error in the income variable can be statistically explained as

$$\text{Plim } (E) = \frac{\sigma_{y_p}^2}{\sigma_{y_p}^2 + \sigma_{y_t}^2}$$

where E is the current income elasticity of the demand for the i th good, $\sigma_{y_p}^2$ is the variance of the permanent income, and $\sigma_{y_t}^2$ is the variance of the transitory income.

elasticity. Therefore, with the inclusion of these variables the estimated income elasticity is lower than with their exclusion.

6. For the estimation technique, our survey of these studies revealed that many authors relied on ordinary least squares for accepting that all the assumptions needed for this technique are upheld. Only a few recognized the existence of problems in the sample that would invalidate the traditional assumptions. For example, Polinsky and Ellwood [39] pointed out that, if the numbers of observations in each city are not the same in a sample, then the mean of the error term is not zero. In this case, they suggested the use of a weighted least squares procedure, where the weights are equal to the square root of the number of observations from each city.

7. For the influence of the supply side, deLeeuw [10] and Vaughn [56] asserted that there is a bias in the estimate of the income elasticity of housing demand as a result of simultaneous interaction between supply and demand in the housing market. Vaughn [56] attributes this bias to the upward housing supply curve that faces a group of buyers in the housing market. For ungrouped buyers, Vaughn concluded that there would be no bias in the income elasticity as long as "individuals face an infinite supply price elasticity" [56, p. 49]. Nonetheless, no generalization seems warranted about the effect of the supply side on the estimate of the income elasticity: the difference

between the estimates of the elasticity based on both kinds of buyers is a matter of empirical investigations.

Other explanations can be given for the differences in the estimate of the income elasticity of the demand for housing reported in Table 1. For example, limitations in the sample, the omission of imputed rent that owner-occupiers receive from their homes, and the use of different functional forms in estimating the housing demand function.

Accordingly, no unified model of the household behavior with regard to consumption of and investment in housing has emerged either as the basis for, or as the result of these studies. In fact, the purposes of this review are not to present such a model, but to examine past research in housing for evidence of the utilization of portfolio analysis and wealth to explain housing demand, to illustrate the differences between their results and those of other studies, and to highlight the need for a coherent theoretical framework that explains housing demand in relation to portfolio behavior and wealth of the household.

This chapter is organized as follows. Section II.2 shows what consideration was made of portfolio analysis in the housing literature. Evidence is presented to show that many authors have utilized portfolio analysis in order to explain their results. Section II.3 presents a summary of the previous attempts at incorporating the wealth effect and the portfolio balance effect into the housing literature.

Finally, Section II.4 explains the need for a coherent theoretical framework that connects wealth and portfolio analysis with the demand for housing.

II.2 Portfolio Considerations and the Demand for Housing

Early evidence of portfolio considerations in the literature of housing is found in Aaron [1] and Goode [16] with their treatment of imputed rent derived from owner-occupied houses. They attempted to investigate the revenue effect of the failure to tax imputed rent on owner-occupied houses under different tax proposals and to see to what extent taxes would affect the portfolio composition of the household.

Aaron [1] visualizes the household as both a consumer and an investor. Changes in the relative costs of owning and renting caused by taxes would change the household-consumer demand for home ownership. Changes in returns on housing equities relative to after-tax returns on other assets caused by taxes would change the household-investor demand for home ownership and hence its portfolio composition. The way the household shuffles its portfolio, according to Aaron, depends upon the tax effect on the relative returns on the asset holdings, and also upon the elasticities of the demand for and supply of housing.

Aaron also investigated the availability of mortgage funds. He held that as home ownership is favored by taxation, it seems more favorable as one moves up the income scale, and hence a high loan-to-value ratio would be desirable. He found that this ratio is negatively correlated with income. Aaron explains this paradox as follows:

First, lending institutions customarily will not lend as large a fraction of price on expensive as on inexpensive houses. Second, high-income taxpayers are probably more likely than low-income taxpayers to have inherited houses owned free and clear. [1, p. 800]

He used total income as the constraint on both consumption and investment choices of the household in the housing market. This might cast some doubt on his results because a significant part of the income variable used in his study is determined by the household asset holdings. Nonetheless, Aaron emphasized that the demand for housing consumption is related to the portfolio structure of the household.

Goode [16] was also concerned with the distributional effects of the special treatment of home ownership under the U.S. income tax laws. He showed, in a way similar to Aaron, that the tax treatment affects the portfolio composition of the household. Housing consumption and its changes due to taxes were emphasized more than in Aaron's study.

Other studies that investigated portfolio consideration in the demand for housing were concerned with the costs of owning versus renting. Shelton [49] postulated that the

economic aspects of the decision to own or to rent depend on the prospective duration of residence of the household in the house. He calculated the ratio of the costs of renting to those of owning (including the income foregone on home owner's investment in the house). He found it to be 11.4 percent for the example he used to analyze the costs of owning versus renting. Shelton concluded that if a landlord cannot get a rent of this level for his property, the landlord would be better off not investing in housing, and hence changing the composition of his portfolio. In addition, Shelton pointed out the importance of the costs of the mobility, credit rating, and down payment requirements of a housing mortgage in determining the household demand for home ownership.

Aside from his numerical calculations, Shelton raised a crucial argument for the inclusion of a portfolio analysis into housing demand studies. It focuses on the way the household choice of owning versus renting should be

evaluated. The choice must not be determined randomly² and its evaluation should account for the portfolio behavior of the household.

II.3 Wealth and the Demand for Housing

Table 1 shows that several previous studies on housing included some demographic variables in their estimates of the housing demand function to control for differences in human wealth among households. The arguments for such an inclusion were mainly built on some investment-in-human capital theories that are concerned with the household behavior over its life cycle. The results of these housing studies were important because they provided the link between the household characteristics, ability to pay, and the housing consumption decision. They also provided the real test for

²Trost [54] analyzed the household choice of owning versus renting according to the principles of utility analysis. He asserted that the demand for housing is jointly determined by the household choice between owning and renting, and how much to spend on housing. He utilized a two-stage maximum likelihood procedure to estimate the demand-for-housing function. In the first stage, he utilized probit analysis to estimate the choice equation where the dependent variable is dichotomous (one for owning, and zero for renting). The choice equation provides the probability of owning. In the second stage he used ordinary least squares to estimate the housing expenditure equation. He argued that the simultaneity between the choice equation and the expenditure equation will bias the ordinary least squares estimates. His estimates of the income elasticity of the demand for housing are 0.55 for owners and 0.50 for renters. His estimates included some proxy variables to capture the differences among households in searching and transaction costs of buying a house.

the stability of the household housing demand over time. These predictors of behavior should be used with caution unless there is a considerable argument for their inclusion in the housing demand function.

For the non-human wealth component, there are a few studies that addressed the effect of the non-human wealth on the household demand for housing. They can be classified, according to their estimation techniques, into two categories: a) those which estimated an ordinary housing demand function, and b) those which estimated a linear-probability housing demand function. Their arguments for the inclusion of wealth and results are summarized below.

For the first category, Arcelus and Meltzer [6] used a macro model to estimate a system of equations of the demand for and supply of housing. Their argument for the inclusion of the net worth into the housing demand function is that the net worth is the constraint on the household choice in the housing market. As the net worth increases, ceteris paribus, the demand for housing increases. That is, home owner's equity is a portfolio allocation decision as the households look at the relative rates of returns in deciding how much of their net worth to hold as equity in housing. Arcelus and Meltzer estimated the housing demand function based on time-series data from different sources for the periods of 1915-1940 and 1948-1968, omitting the years of price and rent control. For the lack of data

on equities in housing and the net worth, they used the amount of debts on housing as a proxy for equities in housing. They rationalized the use of this proxy on the ground that, since the market value of the home is the sum of equities in and debts on housing, equities are negatively correlated with debts. Consequently, they interpreted the negative coefficient for the debt variable as reflecting the positive effect of increased housing equities on housing demand.

Arcelus and Meltzer reported that the permanent income elasticity of the demand for housing is .94. This value is lower than those of time-series studies which did not include the net worth in their estimates. For example, see Muth [35], and Houthakker and Taylor [18]. This might lead to the conclusion that income and net worth are positively correlated, hence the exclusion of the net worth would bias the income elasticity upward. However, one needs to be careful in interpreting such results. If both income and net worth are included in the same demand function, the sum of the respective elasticities measures response to "scale" and may be the appropriate parameter to compare with the income elasticity from traditional estimates.

Another study was conducted by Projector and Weiss [42] who used grouped data to estimate the elasticity of the demand for an asset with respect to the household net worth. They reported the distribution of the household net worth

and estimates of the effect of the size of net worth on the level of asset holdings among different cohorts. They also showed how the use of different functional forms results in different estimates of the demand for an asset. Their estimates of the elasticities of housing demand with respect to net worth are illustrated in Table 2. If one assumes that the net worth elasticity with respect to permanent income is unity, these estimates are very similar to those permanent income elasticities of housing demand reported by Lee [29].

TABLE 2
ELASTICITY OF THE HOUSING DEMAND
WITH RESPECT TO THE NET WORTH

Functional Form	Age Groups			
	Head under 35	35-54	55-64	Head 65 and over
Constant Elasticity	1.47	1.18	0.91	0.97
Constant Elasticity Adjusted for Additivity	1.26	1.04	0.75	0.79
Exponential	1.02	0.75	0.63	0.55

Source: Projector and Weiss [42, p. 83]

The second category of studies that considered the effect of non-human wealth on housing utilized a linear-probability housing demand function. This was applied to explaining differences between blacks and whites in the probability of home ownership. The pioneering study of Kain and Quigley [19] showed that racial differences in the probability of home ownership can be explained by differences in the wealth position. They used household savings as a proxy to control for non-human wealth in their estimates. That is a dummy variable in the right-hand side of the equation was used "for prior owner, prior renter, and new households. A fourth category, 'prior tenure unreported', is reflected in the intercept" [19, p. 266]. Consequently, they estimated their housing demand equation twice, both with and without this dummy variable. The differences between the two equations showed that the coefficient for permanent income declined as a result of controlling for the wealth position of the household. This indicates that income and wealth might be positively correlated, hence the exclusion of wealth would bias the income elasticity upward, as we mentioned before.

Kain and Quigley also highlighted the portfolio behavior of the household and the role of wealth as a budget constraint in determining its housing demand. They attributed the low probability of home ownership among blacks to the high cost of search, unwillingness of some banks to finance

home purchases by blacks outside the ghetto, the down payment requirements for housing mortgage, and their ability to pay back such a mortgage. Nonetheless, Kain and Quigley recognized that they could not elaborate further on these differences between blacks and whites for the following reason.

Unfortunately, the sample used in this research shares the deficiency of most other surveys in not including information on household assets and wealth. [19, p. 269]

The work of Birnbaum and Weston [7] provided a test for the Kain and Quigley results and extended their explanations of the effect of wealth differences between blacks and whites on the probability of home ownership. Birnbaum and Weston estimated a linear probability function of housing demand using Survey of Economic Opportunity data that have information on the household assets and wealth. The coefficient for wealth in their estimates was positive and significant. That is, the probability of home ownership, hence the demand for housing, would increase with the level of wealth and asset holdings, ceteris paribus. Birnbaum and Weston also estimated two housing demand equations: one with and the other without wealth. Their results showed that the inclusion of wealth reduces the coefficient for permanent income.

A comparison between Birnbaum and Weston [7] and Kain and Quigley [19] reveals that both have attempted to connect the portfolio behavior of the household with its demand for

housing. Both studies showed that housing demand is influenced by the portfolio allocation decision of the household. However, Birnbaum and Weston asserted that the portfolio behavior of blacks is different from that of whites. They estimated two housing demand equations: one for blacks and the other for whites. By applying the Chow test, they rejected the null hypothesis that blacks and whites have a similar portfolio behavior.

II.4 The Need for the Portfolio Analysis in the Housing Demand

This review shows different attempts to explain the complex problem of the income elasticity of housing demand. Each attempt has some merits in predicting the value of the elasticity with respect to the household housing demand in different areas of the U.S.A., at different times, and in different situations. However, the difficulty all these attempts encountered stems from the fact that housing is a multi-dimensional good, and the lack of sufficient data for explaining all the factors that influence this multi-dimensional good. Investigators naturally used what information was available to them for the models they chose to develop in order to estimate the income elasticity. Since the data were limited in the first place, the results they arrived at are understandably tentative and debatable. One reason for this uncertainty is, of course, the choice of the model itself. Another is the probability of error in

measurement of the variables in question. Meanwhile, the attempts to reconcile these results brought to light several issues which formed the grounds of the debate.

The review of the literature showed that the financial aspect of housing demand was emphasized in only a few studies. This led us to suspect that such an aspect might be a source of differences in the estimates of the income elasticity of the demand for housing. Therefore, we investigated two kinds of studies: the first emphasizes the portfolio behavior of the household and the second emphasizes the wealth effect on housing demand. For the first, we found that they gave consideration to the portfolio analysis mainly to show that the housing choice is influenced by the investment decision of the household. For the second, we found that they made crucial arguments for the inclusion of wealth in housing demand function. Their results illustrated that the inclusion of wealth would lower the income elasticity. We attributed this finding to the likelihood that income and wealth are positively correlated, hence the exclusion of wealth would bias the income elasticity upward. In addition, the changes in the income elasticity related to the inclusion of wealth suggest that we investigate the behavior of the sum of income and wealth elasticities as a composite scale elasticity.

Since our findings revealed that the financial aspect of housing demand does matter, it will be our purpose to

provide a model based on portfolio-balance behavior of the household to explain its housing choice.

CHAPTER III
A PORTFOLIO ANALYSIS OF THE DEMAND
FOR HOUSING: THE MODEL

III.1 Introduction

The model analyzes the household demand for housing within the framework of the portfolio-investment decisions that the household makes in order to maximize utility derived from its returns on assets. The analysis is a portfolio-balance approach, where the investment decisions are made with reference to consumption, and the portfolio structure of the household. Both the income account and the balance sheet of the household, therefore, are considered in determining the household choice in the housing market. In this framework, the household is viewed as an investor who seeks the attainment of an optimal mix of the housing stock and consumption of all other goods and services. The composition of such a mix would depend upon the utility function defined over the services provided by that mix, and the consumption function relating the services obtained to the amounts of goods and assets held in the household portfolio. The ease of attaining such a mix, and hence the trade-off between investment in housing and consumption, is constrained by the size of the household wealth.

In addition, some features of the U.S. income tax laws are incorporated into the model to capture their effects on the household portfolio choice in the asset market.

In this chapter, we present our theoretical development of the portfolio analysis of the demand for housing. Section III.2 shows the interdependency of the household choice about investment in housing and consumption of all other goods and services, including the services provided by the other assets in the portfolio structure of the household. Section III.3 provides the Kuhn-Tucker conditions of the model where an examination for corner solutions would explain the need for the tobit procedure to estimate the housing demand equation. Finally, Section III.4 illustrates the comparative statics of the model, where the signs of the derivatives of the asset demand functions are verified.

III.2 The Interdependency of the Housing Choice

As an investor, the household is faced with alternative ways to allocate its wealth among assets. The way the household holds wealth depends on an array of motives which may be any condition, e.g., increased income now or in the future, a hedge against inflation, a liquid fund for emergencies, shelter provided by its own home, and so on. Assets, in this regard, are held for the income and services they provide to satisfy these motives. However, the allocation of wealth among the different assets takes place

according to costs and yields considerations: the household selects its portfolio composition according to the price of and the returns on assets, subject to the wealth constraint.

Wealth is the constraint on the household demand for assets. It is taken as a given, fixed constant at the time decisions to select the household portfolio are made. At any point in time, wealth is the past accumulation of assets and liabilities: the household can not change this accumulation by shuffling its portfolio composition. However, as time passes the household may make capital gains or save out of current income and change its wealth level, and hence its level of asset holdings.

The model is built in such a way that the household maximizes utility derived from its returns on the housing stock and consumption of all other goods and services, including the services provided by the other assets. The household is assumed to behave according to a utility function, U , which is a function of the housing services provided by the housing stock,¹ Q , and the consumption of all

¹It is important to mention that we make the assumption that the flow of housing services is a function of the housing stock. That is

$$\text{housing services} = h(Q)$$

Therefore the utility function can be written as

$$U = (h(Q), C)$$

Since we assume a monotonic relationship between the housing stock and the housing services, the utility function can also be written as

$$U = (Q, C).$$

other goods and services, C. That is

$$U = u(Q, C) \quad (1)$$

where

$$\frac{\partial U}{\partial Q} = U_1 > 0 \quad (2)$$

$$\frac{\partial U}{\partial C} = U_2 > 0 \quad (3)$$

$$\frac{\partial^2 U}{\partial Q \partial Q} = U_{11} < 0 \quad (4)$$

$$\frac{\partial^2 U}{\partial C \partial C} = U_{22} < 0 \quad (5)$$

$$\frac{\partial^2 U}{\partial Q \partial C} = U_{12} > 0 \quad (6)$$

$$\frac{\partial^2 U}{\partial C \partial Q} = U_{21} > 0 \quad (7)$$

and

$$\frac{\partial^2 U}{\partial Q \partial C} = \frac{\partial^2 U}{\partial C \partial Q} \quad \text{i.e.} \quad U_{12} = U_{21} \quad (8)$$

The first-partial derivatives of the utility function, U_1 and U_2 , are assumed to be positive: the household will

always desire more of both housing services provided by the housing stock and the consumption of all other goods and services. The second-own-partial derivatives, U_{11} and U_{22} , are assumed to be negative: the marginal utilities of the housing services and the consumption of all other goods and services are declining. The cross-partial derivatives, U_{12} and U_{21} , are assumed to be positive: the marginal utility of the housing services is increasing with the consumption of all other goods and services; additionally, the marginal utility of the consumption of all other goods and services is increasing with the housing services. Furthermore, the cross-partial derivatives are assumed to be equal.

The consumption of all other goods and services is determined by the household current income, Y . That is

$$C = kY \quad (9)$$

$$\frac{dC}{dY} = k \quad (10)$$

$$0 < k < 1 \quad (11)$$

The relationship between consumption and the household current income is assumed to be positive, indicating that the marginal propensity to consume, k , is positive. Furthermore, the marginal propensity to consume is assumed to be constant and takes on values between zero and one. Neither

a proportional nor a linear form of the consumption function would alter the analysis. In addition, since the model we present is an equilibrium model, the distinction between current and permanent income is not made, and there is no explicit utility maximization which determines the consumption-saving decision. Therefore, we intend to use a proxy for permanent income in the estimation of the model so as to make our results comparable with the previous estimates of the income elasticity of the demand for housing.

The household current income can be broken down into labor income, Y_ℓ , and income generated by the return on the household asset holdings. This is to distinguish labor income, as an exogenous variable, from the total income which is determined by the household asset holdings. Therefore,

$$Y = Y_\ell + Ar - D.g\left(\frac{D}{PQ}\right) - t(Y_\ell + Ar - D.g\left(\frac{D}{PQ}\right)) \quad (12)$$

where A is the value of the other assets, r is the constant rate of return on the other assets, D is the value of the debt in housing, g is the interest rate on mortgage which is a function of the debt-to-value ratio, $\left(\frac{D}{PQ}\right)$, P is the price of the house, and t is the total income tax payments of the household which is a function of the household taxable income, $(Y_\ell + Ar - D.g\left(\frac{D}{PQ}\right))$.

Equation (12) incorporates the special treatment of the housing stock under the U.S. federal income tax laws. That is, while the returns on all other assets are subject to tax, the imputed rents of the owner-occupied home are excluded from the taxable income. In addition, mortgage interest payments, $D.g(\frac{D}{PQ})$, are allowed to be deducted from the taxable income of the household.

The relationship between income and taxes is assumed to be positive, indicating that the marginal tax rate, t' , is positive. That is

$$\frac{dt(Y_{\ell} + Ar - D.g(\frac{D}{PQ}))}{d(Y_{\ell} + Ar - D.g(\frac{D}{PQ}))} = t' \quad (13)$$

Hence, the taxable income minus the tax payments is

$$Y = (1 - t')(Y_{\ell} + Ar - D.g(\frac{D}{PQ})) \quad (14)$$

Furthermore, to simplify the analysis we have assumed that the relationship between income and taxes is linear: the marginal tax rate is constant. Nonetheless, a nonlinear relationship could be introduced into our model if we were to put up with more complex analyses and conclusions. Meanwhile, our presentation of the income tax would capture the nonproportional relationship between the house value and expenditures on housing services, as advanced by deLeeuw

[10, p. 2]. Such a relationship stems from the fact that the U.S. income tax is progressive.

The idea that wealth is the budget constraint on the household portfolio-allocation decision is based on the fact that the housing asset is large in the sense that the value of the housing stock might be relatively larger than the household wealth. That is, it is not true that the household acquires the housing asset at the moment it purchases the house, unless the value of the house was paid for in cash. Therefore, we define the wealth constraint as the sum of the household assets minus the liabilities secured by these assets.

$$W = PQ + A - D \quad (15)$$

Where W is the net worth of the household, and the other variables are defined as before.

Indeed, if wealth, as defined in equation (15), operates as a budget constraint, it does not directly limit the amount of the housing asset but it limits the household equity in housing. This suggests that the amount of mortgage in housing should be treated as a choice variable which is determined within the model. Therefore, we need to make some assumptions about the availability of the borrowed funds in the mortgage market. We start with the fact that financial institutions always consider the risk of default in their lending operations. Accordingly, we assume

that the interest rate on mortgage is an increasing function of the debt-to-value ratio: the higher the debt-to-value ratio, the higher the probability of the default, and hence the higher the interest rate on mortgage; ceteris paribus. That is

$$\frac{\partial g}{\partial (\frac{D}{PQ})} = g_1 > 0 \quad (16)$$

The second derivative of this function is assumed to be positive: the marginal interest rate on mortgage, g_{11} , is increasing with the debt-to-value ratio. That is

$$\frac{\partial^2 g}{\partial (\frac{D}{PQ}) \partial (\frac{D}{PQ})} = g_{11} > 0 \quad (17)$$

Hence, the slope of the supply curve for the borrowed funds is positive and increasing. In addition, we assume that this slope would asymptotically approach infinity as the debt-to-value ratio approaches unity. The economic rationale for this shape is that the household would be charged a positive interest rate on mortgage if the debt-to-value ratio is less than unity; otherwise there would be no funds available to finance housing purchases. These assumptions guarantee that debt is less than the value of the house, and

this result is crucial for the existence of an interior solution to the model. Accordingly,

$$D < PQ \quad (18)$$

$$\frac{D}{PQ} < 1 \quad (19)$$

therefore,

$$(1 - \frac{D}{PQ}) > 0 \quad (20)$$

Condition (20) explains the economic rationale for the shape of the supply curve for the borrowed funds which makes it a prediction of the model, since D and PQ are determined within the model.

Now we are in a position to derive our model under the assumptions we have made about its variables and their determinants. The problem of the household is to maximize the utility function as defined in equation (1) subject to the wealth constraint as defined in equation (15). Substituting equation (12) into equation (9) yields

$$C = k[Y_\ell + Ar - D.g(\frac{D}{PQ}) - t(Y_\ell + Ar - D.g(\frac{D}{PQ}))] \quad (21)$$

Substituting equation (21) into equation (1) yields

$$U = u(Q, k[Y_\ell + Ar - D.g(\frac{D}{PQ}) - t(Y_\ell + Ar - D.g(\frac{D}{PQ}))]) \quad (22)$$

which defines the household utility function that is to be maximized subject to the wealth constraint as defined in equation (15). The Lagrangian function, L , is

$$L = u(Q, k[Y_{\ell} + Ar - D.g(\frac{D}{PQ}) - t(Y_{\ell} + Ar - D.g(\frac{D}{PQ}))] + \lambda(W - PQ - Ar + D) \quad (23)$$

The first-order conditions are

$$\frac{\partial L}{\partial Q} = U_1 + U_2 k(1 - t')(\frac{D^2}{PQ^2} g_1) - \lambda P = 0 \quad (24)$$

$$\frac{\partial L}{\partial A} = U_2 k(1 - t')r - \lambda = 0 \quad (25)$$

$$\frac{\partial L}{\partial D} = U_2 k(1 - t')(-\frac{D}{PQ} g_1 - g) + \lambda = 0 \quad (26)$$

$$\frac{\partial L}{\partial \lambda} = W - PQ - A + D = 0 \quad (27)$$

hence,

$$\begin{aligned} \frac{U_1}{P} + U_2 k(1 - t')(\frac{D^2}{P^2 Q^2} g_1) &= U_2 k(1 - t')r \\ &= U_2 k(1 - t')(\frac{D}{PQ} g_1 + g) = \lambda \end{aligned} \quad (28)$$

To explain the economic meaning of the equilibrium condition (28), consider the following. Since

$$C = k[Y_\ell + Ar - D.g(\frac{D}{PQ}) - t(Y_\ell + Ar - D.g(\frac{D}{PQ}))] \quad (21)$$

$$\frac{dC}{d(Y_\ell + Ar - D.g(\frac{D}{PQ}))} = k(1 - t') \quad (29)$$

which is the marginal propensity to consume out of the household taxable income. It shows how much consumption would change as the taxable income changes: a one dollar change in the taxable income would result in $k(1 - t')$ dollar change in consumption. Next, consider the term $\frac{D}{PQ} g_1$, which is the loan-to-value ratio, $\frac{D}{PQ}$, times the slope of the mortgage rate function, g_1 . This term is higher the larger the amount of mortgage used to finance housing purchases. On the other hand, had the household taken a smaller mortgage, this term would have presented the gain or the reduction in the costs of a mortgage loan. Accordingly, the equilibrium condition (28) states that the household attains equilibrium when the following are equal: (a) the marginal return on housing, relative to the price of the house, plus the marginal utility of consumption times the net of the interest loan payments that would have been reduced if a smaller amount of housing were financed by a mortgage loan, (b) the marginal utility of consumption times the net of the rate of return on the other assets, (c) the

marginal utility of consumption times the net of the marginal cost of borrowing a mortgage loan, and (d) the marginal utility of wealth.

Apparently, with the introduction of any disturbance, the household can use one or a combination of the following to diversify its portfolio structure, regarding consumption, and hence restore the equilibrium: (1) buy more of the housing asset by selling some of its other assets, (2) borrow more on housing to spend on housing, or (3) borrow more on housing to spend more on the other assets.

III.3 The Kuhn-Tucker Conditions

The equilibrium condition (28) holds if Q , A , D and λ are non-negative values, and the total amount of W is invested in Q and A . If W were not totally invested in Q and A , it would have been possible for the household to invest more in Q , A , or both and thereby increase its income and services provided by assets. The Kuhn-Tucker conditions of the model are

$$\frac{\partial L}{\partial Q} = U_1 + U_2 k(1 - t') \left(\frac{D^2}{PQ^2} g_1 \right) - \lambda P \leq 0 \quad \text{if } Q = 0 \quad (30)$$

$$Q \frac{\partial L}{\partial Q} = Q[U_1 + U_2 k(1 - t') \left(\frac{D^2}{PQ^2} g_1 \right) - \lambda P] = 0 \quad \text{if } Q > 0 \quad (31)$$

$$\frac{\partial L}{\partial A} = U_2 k(1 - t') r - \lambda \leq 0 \quad \text{if } A = 0 \quad (32)$$

$$A \frac{\partial L}{\partial A} = A[U_2 k(1 - t')r - \lambda] = 0 \quad \text{if } A > 0 \quad (33)$$

$$\frac{\partial L}{\partial D} = U_2 k(1 - t') \left(-\frac{D}{PQ} g_1 - g\right) + \lambda \leq 0 \quad \text{if } D = 0 \quad (34)$$

$$D \frac{\partial L}{\partial D} = D[U_2 k(1 - t') \left(-\frac{D}{PQ} g_1 - g\right) + \lambda] = 0 \quad \text{if } D > 0 \quad (35)$$

$$\frac{\partial L}{\partial \lambda} = W - PQ - A + D \leq 0 \quad \text{if } \lambda = 0 \quad (36)$$

$$\frac{\partial L}{\partial \lambda} = \lambda[W - PQ - A + D] = 0 \quad \text{if } \lambda > 0 \quad (37)$$

Conditions (31), (33), (35), and (37) define the case where the household spends all its wealth on Q and A . Accordingly, the household attains the equilibrium position as predicted by the model, and hence such conditions provide an interior solution to the model. However, if Q was not bought, then conditions (30), (33), (34), and (37) would be the necessary conditions for the household equilibrium, and hence such conditions provide a corner solution to the model. Similarly, conditions (31), (32), (35), and (37) would provide another corner solution to the model if A was not bought. For the former case, where $Q = 0$, the wealth constraint is $W = A$, and $\partial Q/dY_\ell$ and $\partial Q/dW$ are zero; while for the latter case, where $A = 0$, the wealth constraint is $W = PQ - D$, i.e. the household equities in housing, and $\partial A/dY_\ell$ and $\partial A/dW$ are zero. Therefore, the examination of

the Kuhn-Tucker conditions indicates that the model would have corner solutions when the household marginal return of a dollar spent on housing is different from the marginal after-tax return of a dollar spent on the other assets. This explains the portfolio-balance behavior of the household where the household may choose to put all its wealth in housing, or all its wealth in the other assets. In addition, for the occurrence of the case where Q is not bought, it might be explained by the fact that the household labor income and wealth have not yet reached the threshold levels that make the household able to pay for housing, or the down-payment requirements of a mortgage to finance housing purchases.

It is worth mentioning that, since all the households in our sample do have some of the other assets, the occurrence of the case where A is not bought has been ruled out. Thus, our investigation into the demand for housing will focus on the cases where $Q > 0$, and $Q = 0$.

Considering the estimation of the housing demand function, with the availability of a sample containing all home owners, ordinary least squares would seem to be the appropriate estimation technique in the sense that all the observation points represent the case where $Q > 0$ and $A > 0$, i.e. the interior solution is held. However, since 34 percent of the households in our sample are non-home owners, i.e. they represent the case for the corner solution where

$Q = 0$, the application of the tobit analysis seems to be the appropriate estimation technique in order to capture these observations off the housing demand curve, and to say something about the decision whether or not to own a house. Therefore, the estimation technique for our housing demand function is the tobit analysis where considerations are given to the existence of such a corner solution in measuring the income elasticity of the demand for housing.

III.4 Comparative Statics of the Model

The rates of change in the decision variables (Q , A , and D) with respect to the parameters (W , Y_g , and P) are derived, in this section, through the use of the implicit function theorem. Therefore, taking the derivatives of the first-order conditions (24)-(27) with respect to these variables, the following results are obtained.

$$\begin{aligned} \frac{\partial^2 L}{\partial Q \partial Q} = & U_{11} + 2U_{12}k(1 - t') \left(\frac{D^2}{PQ^2} g_1 \right) - U_2 k(1 - t') \left(\frac{D^2}{Q^2} v \right) \\ & + U_{22}k^2(1 - t')^2 \left(\frac{D^2}{PQ^2} g_1 \right)^2 \end{aligned} \quad (38)$$

$$\text{where} \quad v = \frac{D}{P^2 Q^2} g_{11} + \frac{2}{PQ} g_1 \quad (39)$$

$$\frac{\partial^2 L}{\partial Q \partial A} = U_{12}k(1 - t')r + U_{22}k^2(1 - t')^2 \left(\frac{D^2}{PQ^2} g_1 \right) r \quad (40)$$

$$\begin{aligned} \frac{\partial^2 L}{\partial Q \partial D} = & - U_{12} k (1 - t') r + U_2 k (1 - t') \left(\frac{D}{Q} v \right) \\ & - U_{22} k^2 (1 - t')^2 \left(\frac{D^2}{PQ^2} g_1 \right) r \end{aligned} \quad (41)$$

$$\frac{\partial^2 L}{\partial Q \partial \lambda} = - p \quad (42)$$

$$\frac{\partial^2 L}{\partial Q \partial W} = 0 \quad (43)$$

$$\frac{\partial^2 L}{\partial Q \partial Y_l} = U_{12} k (1 - t') + U_{22} k^2 (1 - t') \left(\frac{D^2}{PQ^2} g_1 \right) \quad (44)$$

$$\begin{aligned} \frac{\partial^2 L}{\partial Q \partial P} = & U_{12} k (1 - t') \left(\frac{D^2}{PQ^2} g_1 \right) \left(\frac{Q}{P} \right) - U_2 k (1 - t') \left(\frac{D^2}{PQ} v \right) \\ & - \frac{U_1}{P} + U_{22} k^2 (1 - t')^2 \left(\frac{D^2}{PQ^2} g_1 \right)^2 \left(\frac{Q}{P} \right) \end{aligned} \quad (45)$$

$$\frac{\partial^2 L}{\partial A \partial Q} = U_{12} k (1 - t') r + U_{22} k^2 (1 - t')^2 \left(\frac{D^2}{PQ^2} g_1 \right) r \quad (46)$$

$$\frac{\partial^2 L}{\partial A \partial A} = U_{22} k^2 (1 - t')^2 r^2 \quad (47)$$

$$\frac{\partial^2 L}{\partial A \partial D} = - U_{22} k^2 (1 - t')^2 r^2 \quad (48)$$

$$\frac{\partial^2 L}{\partial A \partial \lambda} = - 1 \quad (49)$$

$$\frac{\partial^2 L}{\partial A \partial W} = 0 \quad (50)$$

$$\frac{\partial^2 L}{\partial A \partial Y_\ell} = U_{22} k^2 (1 - t')^2 r \quad (51)$$

$$\frac{\partial^2 L}{\partial A \partial P} = U_{22} k^2 (1 - t')^2 \left(\frac{D^2}{PQ^2} g_1 \right) \left(\frac{Q}{P} \right) r \quad (52)$$

$$\begin{aligned} \frac{\partial^2 L}{\partial D \partial Q} = & - U_{12} k (1 - t') r + U_2 k (1 - t') \left(\frac{D}{Q} v \right) \\ & - U_{22} k^2 (1 - t')^2 \left(\frac{D^2}{PQ^2} g_1 \right) r \end{aligned} \quad (53)$$

$$\frac{\partial^2 L}{\partial D \partial A} = - U_{22} k^2 (1 - t')^2 r^2 \quad (54)$$

$$\frac{\partial^2 L}{\partial D \partial D} = - U_2 k (1 - t') v + U_{22} k^2 (1 - t')^2 r^2 \quad (55)$$

$$\frac{\partial^2 L}{\partial D \partial \lambda} = 1 \quad (56)$$

$$\frac{\partial^2 L}{\partial D \partial W} = 0 \quad (57)$$

$$\frac{\partial^2 L}{\partial D \partial Y_\ell} = - U_{22} k^2 (1 - t')^2 r \quad (58)$$

$$\frac{\partial^2 L}{\partial D \partial P} = U_2 k (1 - t') \left(\frac{D}{P} v \right) - U_{22} k^2 (1 - t')^2 \left(\frac{D^2}{PQ^2} g_1 \right) \left(\frac{Q}{P} \right) r \quad (59)$$

$$\frac{\partial^2 L}{\partial \lambda \partial Q} = -P \quad (60)$$

$$\frac{\partial^2 L}{\partial \lambda \partial A} = -1 \quad (61)$$

$$\frac{\partial^2 L}{\partial \lambda \partial D} = 1 \quad (62)$$

$$\frac{\partial^2 L}{\partial \lambda \partial \lambda} = 0 \quad (63)$$

$$\frac{\partial^2 L}{\partial \lambda \partial W} = 1 \quad (64)$$

$$\frac{\partial^2 L}{\partial \lambda \partial Y_\ell} = 0 \quad (65)$$

$$\frac{\partial^2 L}{\partial \lambda \partial P} = -Q \quad (66)$$

Using Cramer's rule to solve for the multipliers, we start with the calculation of the determinant of the system, Δ . As shown in Figure 1 where the case of W is presented, the determinant of the system is

$$\Delta = U_1^k (1 - t') \left(\frac{D}{P^2 Q^2} g_{11} + \frac{2}{PQ} g_1 \right) \\ (U_1 U_{22} / U_2 - 2 U_{12} + U_2 U_{11} / U_1) < 0 \quad (67)$$

$$\begin{array}{c}
 \left[\begin{array}{l}
 u_{11} + 2 u_{12} k (1 - t') \left(\frac{D^2}{PQ^2} g_1 \right) \\
 - u_2 k (1 - t') \left(\frac{D^2}{Q^2} v \right) \\
 + u_{22} k^2 (1 - t')^2 \left(\frac{D^2}{PQ^2} g_1 \right)^2
 \end{array} \right] \quad
 \left[\begin{array}{l}
 u_{12} k (1 - t') r \\
 + u_{22} k^2 (1 - t')^2 \left(\frac{D^2}{PQ^2} g_1 \right) r \\
 - u_{22} k^2 (1 - t')^2 \left(\frac{D^2}{PQ^2} g_1 \right) r
 \end{array} \right] \quad
 \left[\begin{array}{l}
 -P \\
 -P \\
 -P
 \end{array} \right] \quad
 \left[\begin{array}{l}
 \frac{dQ}{dW} \\
 \frac{dQ}{dW} \\
 \frac{dQ}{dW}
 \end{array} \right] \quad
 \left[\begin{array}{l}
 0 \\
 0 \\
 0
 \end{array} \right] \\
 \\
 \left[\begin{array}{l}
 u_{12} k (1 - t') r \\
 + u_{22} k^2 (1 - t')^2 \left(\frac{D^2}{PQ^2} g_1 \right) r
 \end{array} \right] \quad
 \left[\begin{array}{l}
 u_{12} k (1 - t') r \\
 + u_{22} k^2 (1 - t')^2 \left(\frac{D^2}{PQ^2} g_1 \right) r
 \end{array} \right] \quad
 \left[\begin{array}{l}
 -1 \\
 -1
 \end{array} \right] \quad
 \left[\begin{array}{l}
 \frac{dA}{dW} \\
 \frac{dA}{dW}
 \end{array} \right] \quad
 \left[\begin{array}{l}
 0 \\
 0
 \end{array} \right] \\
 \\
 \left[\begin{array}{l}
 -u_{12} k (1 - t') r \\
 + u_2 k (1 - t') \left(\frac{D}{Q} v \right) \\
 - u_{22} k^2 (1 - t')^2 \left(\frac{D^2}{PQ^2} g_1 \right) r
 \end{array} \right] \quad
 \left[\begin{array}{l}
 -u_2 k (1 - t') v \\
 + u_{22} k^2 (1 - t')^2 r^2
 \end{array} \right] \quad
 \left[\begin{array}{l}
 1 \\
 1
 \end{array} \right] \quad
 \left[\begin{array}{l}
 \frac{dD}{dW} \\
 \frac{dD}{dW}
 \end{array} \right] \quad
 \left[\begin{array}{l}
 0 \\
 0
 \end{array} \right] \\
 \\
 \left[\begin{array}{l}
 -P \\
 -1
 \end{array} \right] \quad
 \left[\begin{array}{l}
 1 \\
 1
 \end{array} \right] \quad
 \left[\begin{array}{l}
 0 \\
 0
 \end{array} \right] \quad
 \left[\begin{array}{l}
 \frac{d\lambda}{dW} \\
 \frac{d\lambda}{dW}
 \end{array} \right] \quad
 \left[\begin{array}{l}
 -1 \\
 -1
 \end{array} \right]
 \end{array}$$

Figure 1. The comparative statics of the model: The case of wealth.

$$\text{let } H = (U_1 U_{22} / U_2 - 2 U_{12} + U_2 U_{11} / U_1) \quad (68)$$

then,

$$\Delta = U_1 k(1 - t')VH < 0 \quad (67')$$

That is, the sufficient second-order condition of the model is that the bordered Hessian determinant of the second partials of L be negative. The assumption that $U_{12} > 0$, $g_1 > 0$, and $g_{11} > 0$ will fulfill this condition.

In fact, the assumptions we made about the g function has made the second-order condition to be fulfilled. Suppose, on the contrary, we were assuming that the household can borrow unlimited funds, in the mortgage market, at a fixed rate. This would have implied that (a) some households would have debt in an amount greater than the value of the house, (b) the system would be undetermined, or (c) wealth would not operate as a budget constraint on anything. Case (a) is unrealistic, and cases (b) and (c) do violence to our model where Δ would be zero as a result of $g_1 = g_{11} = 0$. That is, the assumptions made about the g function are crucial for the derivation of our results.

However, using Cramer's rule again for the cases of Y_λ and P , the following results of the comparative statics of the model are taken.

$$\frac{dQ}{dW} = \frac{1}{H} [rk(1 - t')(U_{22} - U_{12}U_2/U_1)] > 0 \quad (69)$$

$$\frac{dA}{dW} = 1 - \frac{1}{H} [rk(1 - t')P(1 - \frac{D}{PQ})(U_{22} - U_{12}U_2/U_1)] > 0 \quad (70)$$

$$\frac{dD}{dW} = \frac{1}{H} [rk(1 - t')\frac{D}{Q}(U_{22} - U_{12}U_2/U_1)] > 0 \quad (71)$$

$$\frac{dQ}{dY_\ell} = \frac{1}{H} [k(1 - t')(U_{22} - U_{12}U_2/U_1)] > 0 \quad (72)$$

$$\frac{dA}{dY_\ell} = \frac{-1}{H} [k(1 - t')P(1 - \frac{D}{PQ})(U_{22} - U_{12}U_2/U_1)] < 0 \quad (73)$$

$$\frac{dD}{dY_\ell} = \frac{1}{H} [k(1 - t')\frac{D}{Q}(U_{22} - U_{12}U_2/U_1)] > 0 \quad (74)$$

$$\frac{dQ}{dP} = \frac{-1}{H} [\frac{-U_2}{P} + \frac{Q}{P}(U_{22}U_1/U_2 - U_{12})] < 0 \quad (75)$$

$$\frac{dA}{dP} = \frac{1}{H} [Q(1 - \frac{D}{PQ})(U_{12} - U_2U_{11}/U_1 - U_2/Q)] \gtrless 0 \quad (76)$$

$$\frac{dD}{dP} = \frac{1}{H} [\frac{D}{P}(U_{11}U_2/U_1 + U_2/Q - U_{12})] \gtrless 0 \quad (77)$$

Therefore, there exist three asset demand equations in our model. These equations and their derivatives with respect to W , Y_ℓ , and P are the following.

(1) The housing demand equation

$$Q = q(W, Y_{\ell}, P) \quad (78)$$

where

$$\frac{\partial Q}{\partial W} > 0 ; \frac{\partial Q}{\partial Y_{\ell}} > 0 ; \text{ and } \frac{\partial Q}{\partial P} < 0$$

(2) the other-assets demand equation

$$A = s(W, Y_{\ell}, P) \quad (79)$$

where

$$\frac{\partial A}{\partial W} > 0 ; \frac{\partial A}{\partial Y_{\ell}} < 0 ; \text{ and } \frac{\partial A}{\partial P} \geq 0$$

(3) the demand for housing mortgage equation

$$D = e(W, Y_{\ell}, P) \quad (80)$$

where

$$\frac{\partial D}{\partial W} > 0 ; \frac{\partial D}{\partial Y_{\ell}} > 0 ; \text{ and } \frac{\partial D}{\partial P} \geq 0$$

It is worth mentioning that our results indicate that

$$\frac{\partial Q}{\partial W} = r \frac{\partial Q}{\partial Y_l} \quad (81)$$

or

$$\frac{\partial Q}{\partial W} / \frac{\partial Q}{\partial Y_l} = r \quad (82)$$

That is, the estimation of the housing demand function would provide a measurement of the magnitude of the rate of return on A by dividing the wealth coefficient in the housing demand function over the labor income coefficient in the same function. Similarly, this result holds for the other two demand equations.

Thus far, we are able to derive a model that is built on the portfolio behavior of the household in the assets markets regarding the consumption decision of the household. Assets were visualized as parts of the household wealth and as the source of services provided to the household. Our analysis has many merits similar to those found in the frameworks which analyzed the relationship between consumption and the portfolio structure of the household. That is, the model suggests that the household portfolio-allocation decision would depend not only on the investment returns obtainable on assets but also on the household demand for

consumption of services provided by the housing stock. We defined a utility function over the services provided by the mix of consumption and the amount of the housing stock. We tied the consumption function and the mortgage rate function with the model in order to explain the relationship between consumption and income generated by asset holdings, and to explain the decision of how much to borrow as a portfolio decision, respectively. With the wealth constraint and the credit constraint, we were able to derive the first-order conditions which showed the equilibrium conditions of the household regarding consumption and investment simultaneously. Upon the derivation of these conditions, it was noticed that a change in a parameter results in changes in the amount of the housing stock, and hence the housing services, and changes in the consumption of all other goods and services, including the services provided by the other assets. This explains the trade-off the household makes between consumption and its level of asset holdings. The model explains the wealth effect on such a trade-off by the assertion that there would be some down payments to be made by the household in order to get mortgage credits. That is, there is a threshold level which the household wealth has to reach for the provision of home ownership. In addition, the model includes the different features of the U.S. federal income tax laws which were found to be effective in the determination of the household choice in the housing market:

changes in the marginal tax rates would alter the household choice in the housing market, as condition (28) indicates.

In fact, the housing demand function (78) derived from our model is different from its counterparts in the previous housing studies because it includes wealth. Condition (69) indicates that $\partial Q / \partial W$ would be zero if the marginal utility of consumption and, hence, of income was constant. This is an implicit assumption of the past housing studies. On the other hand, as we have shown upon the calculation of the determinant of the system, Δ , such a system of equations would be undetermined if we were assuming that the interest rate on mortgage is fixed. Such a result would have emerged from the equality of g and r , hence, the second row of the determinant, Δ , would provide the same information in the third row of the determinant, and the value of the determinant would have been zero. These two assumptions would have led us to drop wealth from our housing demand function (78), if they were assumed in our model. However, there are some evidences in the economic literature which show that our assumption about the marginal utility of income is valid, and our assumption about the interest rate on mortgage is more realistic, in explaining credit rationing and the existence of stringent factors that limit the household ability to borrow in the mortgage market, than the fixed interest rate assumption.

For the assumption about the marginal utility of income, Samuelson [48] showed that the marginal utility

is not always necessarily constant, but it could be of any magnitude and sign. He held that the marginal utility of income is a function of income and prices of goods in the consumption bundle of the household. This function is homogeneous of degree minus one in income and prices. That is, a doubling in income and prices would halve the marginal utility of income, and leave the quantity of goods unchanged. With the examination of different cases and conditions, Samuelson concluded that

. . . by an appropriate selection of utility index the rate of change in the marginal utility of income with respect to any parameter can be made to be of any arbitrary magnitude and sign. [48, p. 77]

In addition, Friedman and Savage [14] used some empirical observations to analyze the individual's income choices involving risk among the different income classes. Their conclusion about the individual's preference, in this context, is that most individuals tend to have incomes that place them in the segments of the utility function where the marginal utility of income is diminishing.

For the assumption of the fixed interest rate on mortgage, several authors, like Muth [36], Lee [28], Fair [12], and Kearn, Rosen, and Swan [22], have investigated the relationship between the mortgage rate and the demand for housing. They held that the fixed interest rate on mortgage is not a complete measure of the availability of mortgage credit in the mortgage market, and hence other credit-term variables, such as the loan-to-value ratio and the length of

the contract, should be considered in explaining the determination of the interest rate on mortgage. That is, they suggested a modification of some of the behavioral assumptions underlying the determination of that interest rate in such a way as to capture the effects of these factors on the demand for housing. Accordingly, the assumption that the interest rate on mortgage is an increasing function of the debt-to-value ratio, as it was made in our model, seems to be more realistic in considering the effects of such factors. Besides, as was shown in the equilibrium condition (28), it allows for the household to vary the amount of the initial housing equities via the size of the down payments on housing as a portfolio-allocation decision.

Finally, our model suggests that it is useful to look at wealth and labor income, rather than simply the total income, as the determinants of the demand for an asset. For the housing demand function, as an example, one can calculate the change in Q per dollar change in total income, Y , when W changes by

$$\frac{dQ}{dY} = \frac{dQ}{dW} / \frac{dY}{dW} \quad (83)$$

where

$$\frac{dY}{dW} = \frac{d}{dW}(Y_L + Ar) = r \frac{dA}{dW}$$

In addition, one can calculate the change in Q per dollar change in Y , when Y_ℓ changes by

$$\frac{dQ}{dY} = \frac{dQ}{dW} / \frac{dY}{dY_\ell} \quad (84)$$

where

$$\frac{dY}{dY_\ell} = \frac{d}{dY_\ell}(Y_\ell + Ar) = 1 + r \frac{dA}{dY_\ell}$$

Using results (69) and (70) to calculate the term in (83), and using results (72) and (73) to calculate the term in (84), we have found that there is no difference between our method in measuring the changes in Q due to changes in Y_ℓ and W ; and other methods that used only the total income. Accordingly, the income elasticity of the demand for housing is the sum of the elasticities of the demand for housing with respect to labor income and wealth, that is, equal percentage changes in labor income and wealth produce the same percentage change in total income. This useful aspect of our model is due to the distinction we made between the labor income, as an exogenous variable, and the total income, which is determined by the household asset holdings. Therefore, the model suggests the regression of Q on Y_ℓ and W as the exogenous variables to capture the magnitude and the direction of the change in each variable separately. Besides, it would enable us to avoid simultaneous equations

bias in the estimation of the housing demand function. This aspect will be investigated in the next chapter.

CHAPTER IV

ESTIMATION OF THE HOUSING DEMAND FUNCTION

IV.1 Introduction

A major important question related to the housing demand is how responsive demand is to changes in income. A part of the answer to this question, as suggested by our model, is to consider the relationship between consumption and the portfolio structure of the household. In this framework the emphasis is given to labor income and wealth as the main determinants of such a degree of responsiveness. Labor income determines the marginal utility of consumption versus the marginal utility of additional housing services, while wealth is a budget constraint on the household choice in the housing market. The distinction between labor income and total income indicates the need for a further investigation of the way the previous studies on housing have looked at the income elasticity of the demand for housing. The other part of the answer is empirical in nature, that is, the treatment of the zero holdings of the housing stock, in the sample, where the non-negativity constraint is often binding. Thus, elimination of the non-home owners (renters) excludes useful information that can be utilized to explain the decision whether or not to own

a house, while their inclusion in the same manner as those households that own houses might not qualify for the assumptions of any standard estimation technique, such as ordinary least squares, especially if there were a large number of observations. Therefore, the use of the tobit procedure is the appropriate estimation technique where not only the size of responses is considered in the estimates of the elasticities of the demand for housing but also the probability of holding the housing stock in the portfolio structure of the household. With the utilization of such probabilities, this procedure provides better measurements for the prediction of the elasticities of the demand for housing than any other procedure that separates the sample data into owners and renters and estimates two demand equations. Thus, in this approach, as it is explained in both parts, we hope to come up with the right answer to the question of the income elasticity of the demand for housing.

The organization of this chapter will be as follows. Section IV.2 presents the housing demand function as we derived it in the previous chapter, with more emphasis on its features compared with its counterparts in past housing studies. In section IV.3 we specify our housing demand function as we estimate it. Finally, in section IV.4 we explain the estimation technique, the tobit analysis, we use to estimate our housing demand function. In addition, a test for homoscedasticity is given in

order to rationalize the use of a heteroscedastic tobit analysis to estimate our housing demand function.

IV.2 The Housing Demand Function

The results of the comparative statics of the model show that there exists a housing demand function such as

$$Q = q(W, Y_h, P) \quad (78)$$

The derivatives of this function with respect to W , Y_h , and P , as indicated by conditions (69), (72), and (75), respectively, predict that the quantity of the housing demanded would increase when (a) the household wealth increases, (b) the household labor income increases, and (c) the price of the house falls, ceteris paribus.

Wealth enters the housing demand function (78) as the budget constraint on the household choice in the housing market. It affects the trade-off between consumption and the portfolio structure of the household. That is, as condition (69) indicates, changes in wealth will result in changes not only in the portfolio structure of the household but also the consumption decision of the household. The higher the demand for housing, ceteris paribus, the larger the size of the household wealth. Wealth also is an indicator of the ability of the household to meet the down-payment requirements of a mortgage loan on housing. In addition, the marginal propensity to consume

taxable income, $k(1-t')$, in (69) indicates that such a trade-off would be influenced by the magnitude of the tax rate of the household, t' .

Labor income of the household enters the housing demand function (78) as a determinant of the marginal utility of consumption versus the marginal utility of additional housing services. Changes in labor income lead to changes in consumption and the portfolio structure of the household, as the equilibrium condition (28) indicates. If the household desires to maintain its consumption level, for instance, it should adjust its portfolio structure in a way that is compatible with the changes in the labor income. On the other hand, as condition (72) shows, the labor-income coefficient would capture the effects of the tax exemptions and tax deductions of the home ownership on the trade-off between consumption and the portfolio structure of the household. The higher the marginal tax of the home owner, the larger the exemptions and the deductions would be, and hence the higher the demand for housing, ceteris paribus.

Price of the house enters the housing demand function (78) as a determinant of the household choice in the housing market. It is the cost of the housing services provided by the housing stock. As the price of the house increases, for example, the relative prices of the other goods would change, and hence the consumption decision of the household. Consequently, this would alter the portfolio structure of the household, and the household would be willing to

purchase less housing because of the relatively cheaper goods and assets in its consumption bundle and its portfolio, respectively.

For the estimation of the housing demand function, we will use the data of the Survey of Financial Characteristics of Consumers [42], which was conducted in the spring of 1963, to estimate the housing demand function (78). Since, as in most housing studies, there are no data available to measure Q and to estimate such a function, we will use the market value of the home as the dependent variable. In fact, the use of the market value would not alter the measurements of the elasticities of the demand for housing. We can show this by the following.

$$\text{Let} \quad M = PQ \quad (85)$$

where M is the market value of the home, and the other variables are defined as before. Taking the derivative of (85) with respect to Q

$$\frac{\partial M}{\partial Q} = P \quad (86)$$

Therefore,

$$\frac{\partial M}{\partial W} \frac{W}{M} = \left(\frac{\partial M}{\partial Q} \frac{\partial Q}{\partial W} \right) \frac{W}{M} = \left(P \frac{\partial Q}{\partial W} \right) \frac{W}{PQ} = \frac{\partial Q}{\partial W} \frac{W}{Q} \quad (87)$$

Similarly,

$$\frac{\partial M}{\partial Y_\ell} \frac{Y_\ell}{M} = \frac{\partial Q}{\partial Y_\ell} \frac{Y_\ell}{Q} \quad (88)$$

$$\frac{\partial M}{\partial P} \frac{P}{M} = 1 + \frac{\partial Q}{\partial P} \frac{P}{Q} \quad (89)$$

That is, the estimation of

$$M = m(W, Y_\ell, P) \quad (90)$$

and the calculation of the elasticities out of (90) will provide us with the values of the elasticities of equation (78).

At this point two comments are in order regarding the relationship between our estimates of (90) and previous empirical works. First, our use of market value will likely lead to higher estimated income elasticities than those derived from the use of housing expenditures as the left-hand side variable. Typically, expenditures rise less in proportion as market value rises. Second, if data on prices of housing were available, we would both deflate M , in order to make Q (the left-hand side variable) and include P as an additional regressor as had sometimes been done in other studies.

Our results are not strictly comparable with other studies, however. It is the difference in results for

different ways of estimating (90) which is of primary interest, not a comparison of our estimates of (90) with previous estimates.

Using the multipliers in (69), (72), and (75), equations (87)-(89) can be rewritten as

$$\frac{\partial M}{\partial W} \frac{W}{M} = \frac{W}{MH} [rk(1 - t')(U_{22} - U_{12}U_2/U_1)] > 0 \quad (87')$$

$$\frac{\partial M}{\partial Y_\ell} \frac{Y_\ell}{M} = \frac{Y_\ell}{MH} [k(1 - t')(U_{22} - U_{12}U_2/U_1)] > 0 \quad (88')$$

$$\frac{\partial M}{\partial P} \frac{P}{M} = 1 + \frac{1}{MH} [U_2 - Q(U_{22}U_1/U_2 - U_{12})] < 0 \quad (89')$$

The elasticity coefficients in (87')-(89') indicate that each coefficient is a function of the variables and parameters of our model. Therefore, equation (90) must be specified in such a way that the functional form captures the non-linear characteristics of W , Y_ℓ , and P . Accordingly, expanding equation (90) by Taylor-series expansion (see Heady and Dillon [17, p. 204])

$$M = \beta_0 + \beta_1 Y_\ell + \beta_2 Y_\ell^2 + \beta_3 W + \beta_4 W^2 + \beta_5 Y_\ell W + \xi \quad (91)$$

Since our model predicts that the labor-income elasticity of the demand for housing is positive, from (91) the term

$$(\beta_1 + 2\beta_2 Y_\ell + \beta_5 \frac{Y_\ell}{M}) \quad (92)$$

would be positive. And also for the wealth elasticity, the term

$$(\beta_3 + 2\beta_4 W + \beta_5 \frac{Y_\ell}{M}) \frac{W}{M} \quad (93)$$

would be positive.

In fact, the formulation of the housing demand function as in (91) is different from its counterparts in past housing studies. Many of these studies have treated the housing good as a consumption good where the price of the house (or rent), prices of the other goods, and income are the relevant arguments in the demand function. In such treatments there was a separation between asset-acquisition decision and service-consumption decision of the household demand for housing. In contrast, our model treats housing as an asset in the portfolio structure of the household that can be purchased to provide a flow of consumption services, as returns on such an asset, to its owner. Thus, the decision whether or not to own a house has the features of both approaches: the first emphasizes the relationship between the portfolio structure of the household and the household's demand for the housing asset, while the second emphasizes the relationship between consumption and income generated by the household asset holdings.

Another difference is the way in which the model explains the determination of the household income. In past studies of housing that emphasized the first approach, current income or permanent income is the budget constraint on the household choice in the housing market. Particularly important is the explicit endogeneity of income and the possibility of not explicitly considering the simultaneity problem.¹ That is, the assumption that income is predetermined may be unrealistic, and thereby cause an underestimation of the income coefficient in the housing demand function. As we have shown in our model, a distinction should be made between labor income, as exogenously determined, and total income which is determined by the household asset holdings. Therefore, we expect to avoid the simultaneity problem in our estimation of the housing demand function, where both labor income and wealth are exogenously determined. A test for such an argument can be provided to show that the use of total income, under our model's assumptions, produces a simultaneous equations bias in the estimates of the income elasticity of the demand for

¹Trost [54] has dealt with a somewhat different context of the simultaneity problem of the second approach.

housing. As an illustration, consider the linear version of a system of equations of the demand for assets.

$$M_i = PQ = \alpha_0 + \alpha_1 Y_{\ell_i} + \alpha_2 W_i + \epsilon_{i1} \quad (94)$$

$$A_i = b_0 + b_1 Y_{\ell_i} + b_2 W_i + \epsilon_{i2} \quad (95)$$

where

$$M + A = W + D \quad (96)$$

is an identity. In this system the role of wealth was justified as a behavioral determinant of the demand for an asset. Therefore, the estimation of this system or any of its demand equations, with the use of wealth as a regressor, would avoid the simultaneity problem in the sense that wealth is not correlated with any of the error terms in (94) and (95). If we use total income, Y , as it was defined in the past housing studies to estimate the housing demand function as

$$M_i = \gamma_1 Y_i + \epsilon_{i3} \quad (97)$$

where

$$Y = Y_{\ell} + rA \quad (98)$$

substituting (95) into (98) yields the determination of the household total income under our model's assumptions.

$$Y_i = b_0 r + (1 + b_1) r Y_{i1} + b_2 r W_i + r \epsilon_{i2} \quad (99)$$

In this case, one might suspect a relationship between ϵ_{i1} and ϵ_{i2} stemming from the simultaneous trade-off between the holdings of the housing stock and the other assets in the portfolio structure of the household because the holding of the housing stock requires the holding of a relatively lower level of the other assets, since wealth is assumed to be fixed. That is, ϵ_{i1} and ϵ_{i2} are negatively correlated, and as such would bias the ordinary least squares estimates of the income elasticity of the demand for housing. Using ordinary least squares to estimate (97), the slope parameter estimate is

$$\hat{\gamma}_1 = \frac{\sum Y_i M_i}{\sum Y_i^2} \quad (100)$$

Substituting (97) into (100) yields

$$\hat{\gamma}_1 = \frac{\sum Y_i (\gamma_1 Y_i + \epsilon_{i3})}{\sum Y_i^2} \quad (101)$$

Taking the expected value of (101)

$$E(\hat{\gamma}_1) = \gamma_1 + \frac{\text{Cov}(Y_i, \epsilon_{i3})}{\sigma_{Y_i}^2} \quad (102)$$

where the "hat" above the coefficient refers to its estimated value, $\text{Cov}(Y_i, \epsilon_{i3})$ is the covariance of Y_i and ϵ_{i3} , and $\sigma_{Y_i}^2$ is the variance of Y_i . The numerator of the second term in the right-hand side of (102) can be calculated with the use of (99) as the following.

$$\begin{aligned} \text{Cov}(Y_i, \epsilon_{i3}) &= E(Y_i \epsilon_{i3}) \\ &= E([b_0 r + (1 + b_1) r Y_{\ell_i} + b_2 r W_i + r \epsilon_{i2}] \epsilon_{i3}) \end{aligned} \quad (103)$$

Since Y_{ℓ} and W are non-stochastic, and the expected value of the error term, ϵ_{i3} , is assumed to be zero,

$$\text{Cov}(Y_i, \epsilon_{i3}) = r \sigma_{\epsilon_{i2}, \epsilon_{i3}} \quad (104)$$

where $\sigma_{\epsilon_{i2}, \epsilon_{i3}}$ is the covariance of ϵ_{i2} and ϵ_{i3} . Hence the term in (102) can be rewritten as

$$E(\hat{\gamma}_1) = \gamma_1 + \frac{r \sigma_{\epsilon_{i2}, \epsilon_{i3}}}{\sigma_{Y_i}^2} \quad (105)$$

Therefore, since $\sigma_{\epsilon_{i2}, \epsilon_{i3}}$ is assumed to be negative, the use of the total income as a regressor in the housing demand function would produce an inconsistent estimate of the true income coefficient, and hence such a coefficient would be biased. The direction of the bias is downward, and the degree of underestimation depends upon r and $\sigma_{\epsilon_{i2}, \epsilon_{i3}} / \sigma_{y_i}^2$. Accordingly, we will use a two-stage procedure so as to purge the total income variable in order to estimate the housing demand function. This result will be illustrated in the next chapter where we compare our results with those of the past studies on housing.

The last difference between our housing demand function and its counterparts in the past housing studies is the inclusion of wealth in the function. Most of the earlier studies have excluded a wealth variable due to the lack of data. This might have produced a bias in their estimates of the income elasticity of the demand for housing due to the likelihood that wealth and total income are positively related: wealthier households tend to have relatively higher total income than others. Accordingly, the exclusion of wealth would overestimate the income elasticity of the demand for housing. Such a result, can be shown by considering the linear version of a true model that explains the housing demand function as

$$Q_i = a_1 Y_i + a_2 W_i + \epsilon_{i1}^* \quad (106)$$

If one estimates

$$Q_i = a_1^* Y_i + \varepsilon_{i1}^* \quad (107)$$

with the use of ordinary least squares

$$\hat{a}_1^* = \frac{\sum Y_i Q_i}{\sum Y_i^2} \quad (108)$$

Substituting Q_i as defined in (106) into (108) and solving

$$\hat{a}_1^* = \frac{\sum a_1 Y_i^2 + \sum a_2 Y_i W_i + \sum Y_i \varepsilon_{i1}^*}{\sum Y_i^2} \quad (109)$$

Since Y and W are non-stochastic and the expected value of ε_{i1}^* is assumed to be zero, the expected value of the parameter estimate is

$$E(\hat{a}_1^*) = a_1 + a_2 \frac{\text{Cov}(Y_i, W_i)}{\sigma_{Y_i}^2} \quad (110)$$

where $\text{Cov}(Y_i, W_i)$ is the covariance of Y and W . The numerator of the second term in the right-hand side of (110) is positive under the assumption that wealth and total income are positively related. Thus, as (110) indicates, the exclusion of wealth would overestimate the true income coefficient of the housing demand function. The degree of

overestimation would depend upon a_2 times the ratio of the $\text{Cov}(Y_i, W_i)$ to the variance of Y_i . This result will also be illustrated in the next chapter.

IV.3 Specification of the Housing Demand Function

The housing demand function in (91) will be estimated on a sample of 1,984 households from the Survey of Financial Characteristics of Consumers [42], henceforth the "Survey." The sample includes information about the portfolio structure of the households as of December 31, 1962, and the income reported was the before-tax income received during 1962. These households were interviewed again in the spring of 1964 to obtain data on their income and saving during 1963 on a somewhat similar basis. Data of wealth published in the Survey incorporate corrections that the households made when they were reinterviewed for the saving survey. Data on the household savings and incomes during 1963 were published in the Survey of Changes in Family Finances [41]. In addition, data on the size of the place, regions wherein the households live, age, occupation, and education of the head of the household, and other demographic characteristics were also collected in the Survey. In fact, the original size of the sample was 2,557 households. After excluding those observations that reported negative labor income, and negative wealth or its components the size of the sample

became 1,984. Information on these 1,984 households was also available in the saving survey of 1963. Such information will be used to calculate the average labor income of 1962 and 1963, and the average total income of 1962 and 1963 in the estimation of the housing demand function (91). A statistical summary of these data can be found in Appendix 1.

The dependent variable is the market value of the home, M , which is defined as the household's estimate of the market value of its principal residence. This definition does not include owner-occupied farm residences; they were included under business and professions in the sample. According to this definition, 66 percent of the 1,984 households have reported a higher-than-zero market value of the home, while the rest have reported a zero market value. The zero market value of the home does not mean that the home the household owns has a zero value, but rather it means that the household owns no home. It is important to mention that we assume that the response error in the market value of the home exhibits a random variation around the true value, as shown by Kish and Lansing [24], and Kain and Quigley [20]. In addition, the use of the market value of the home in our model does not qualify for deLeeuw's assertion [10] that the estimate of the income elasticity of the demand for housing, based on the use of the market value of home, would be biased upward, simply because we do not use

the market value of home as a proxy for the household expenditures on housing.

The first independent variable is labor income, Y_{ℓ} , which consists of wages and salaries of members of the household, plus net income from sole proprietorships and partnerships.² For a comparison of our study with other housing studies we use total income which consists of labor income, commissions, dividends, interest receipts, net income from rents, pensions and social security payments, and any other periodic payments received by members of the household. For the permanent income concept, we will use the average labor income of 1962 and 1963 as a proxy for the permanent labor income, and the average total income of 1962 and 1963 as a proxy for the permanent income. For the calculation of these averages, we use the same components of the income variables in 1962 and 1963. Upon these calculations, the average labor income, $\bar{Y}_{\ell}$, is defined as the sum of the labor income in 1962, $Y_{\ell 62}$, and labor income in 1963, $Y_{\ell 63}$, divided by two, that is, $\bar{Y}_{\ell} = (Y_{\ell 62} + Y_{\ell 63})/2$. Similarly, the

²This method of calculating labor income is roughly consistent with the concept of the return to labor as a factor of production. One could, alternatively, define labor income as all income which does not depend on portfolio composition. With our data we could accommodate this definition by adding the returns from pensions and annuities (most social security payments) to labor income. We have not performed any estimates based on such a calculation. Since only 25 percent of our households report such income and the mean value is approximately \$400/year, one would not anticipate any substantial changes in results if such a definition were used.

average total income, $\bar{Y}$, is defined as the sum of the total income in 1962, Y_{62} , and total income in 1963, Y_{63} , divided by two, that is, $\bar{Y} = (Y_{62} + Y_{63})/2$. All these income variables were measured before any payroll or income tax deductions.

Another independent variable is the net worth of the household, W , which is defined as the sum of the asset components, less debts secured by these assets. The asset components are the market value of home, automobile, business and professions (farms and nonfarms), liquid assets, investment assets, and miscellaneous assets as reported in the Survey [42].

According to our specification of the functional form in (91), the independent variables Y_{ℓ}^2 , W^2 , and WY_{ℓ} are included in the estimation of the housing demand function. The reason for their inclusion is to test for the non-linear characteristics of Y_{ℓ} and W , and their effects on the demand for housing.

To control for the demographic characteristics of the household and their effects on the household demand for housing, a dummy variable for male is included to capture the effect of sex of the household head, a dummy variable for married household is included to capture the effect of the marital status of the household head, a dummy variable for white is included to capture the effect of the race of the household head, and a dummy variable for self-employed

is included to capture the effect of the occupation of the household head on the household demand for housing. These independent variables are SEX, MART, RACE, and OCC, respectively: where SEX is equal to one when the household head is male, MART is equal to one when the household head is married, RACE is equal to one when the household head is white, and OCC is equal to one when the household head is self-employed; and each variable is equal to zero otherwise. They are included in the estimation of the housing demand function to measure the differences among the households in their preferences towards home ownership, and hence the demand for housing. In addition, their inclusion reduces the over-estimation problems in the elasticities of the demand for housing as shown by Lee [29].

Education of the household head, as the number of years in education; age of the head of the household, as the number of calendar years; age squared; and the household size, as the number of persons in the household are also independent variables included in our estimation of the housing demand equation in (91). They are EDUC, AGE, $(AGE)^2$, and FSIZ, respectively. The reason for their inclusion is to capture the effect of the stages of the household life cycle on the decision of home ownership. In addition, for the household size, it is expected that larger household units should lead to a larger marginal rate of services provided by the housing stock. Nonetheless, all these

demographic variables will be used to estimate a first-stage equation, where a predictor of the permanent labor income and a predictor of the permanent income will be generated. These are to be used in the second stage, where we compare our estimation of the housing demand function and its results with the past housing studies and their results. Such a procedure would provide the test for the simultaneous-bias problem found in the past housing studies.

The last independent variables included are the size of the place, and region. For the size of the place, there are five different sizes of areas in the sample: urban areas with population of one million or more, urban areas with population between 250,000 and one million, urban areas with population less than 250,000, urban places outside urban areas, and rural areas. For regions, the sample covers four in the United States: Northeast, North Central, South, and the West. Dummy variables for the first three areas, UAP_1 , UAP_2 , and UAP_3 are included in the estimation of the housing demand function (91). In addition, dummy variables for the first three regions, REG_1 , REG_2 , and REG_3 are included in the same function. The reason for including these dummy variables is to control for the effects of the prices of housing which are different from one place to another, and across the regions of the United States.

It is important to notice that the housing price variable has been omitted from our specification of the housing

demand function. The reason is that the Survey does not report any information on such a variable. Meanwhile, the use of other sources of data to construct a housing price index might be irrelevant to our sample data. Therefore, we expect to have some bias in our estimates of the labor income and wealth elasticities of the demand for housing. Such a bias stems from the correlation between labor income, wealth, and the price of housing as shown by several studies on housing, especially Polinsky's [38], which explains the price of housing as a spatial phenomenon, considering the application of the theory of residential location to the demand for housing. Accordingly, within a housing market, wealth, labor income, and prices of housing are negatively related: wealthy and relatively high labor income households tend to purchase more of the housing assets in a location where prices of housing are low. This intracity effect indicates that the omission of the price variable would underestimate both wealth and the labor income elasticities of the demand for housing, under the assumption that both elasticities exceed the elasticities of the marginal commuting costs out of wealth and labor income, respectively. There also is an intercity effect where wealthy and relatively high labor income households are more likely to live in cities with higher averages of wealth, labor income, and prices of housing. This indicates that the averages of these three variables are positively correlated. Thus, the omission of the price variable would

overestimate both elasticities of the demand for housing. Now, for a sample like ours, which was drawn from different areas and regions of the United States, the bias in the estimates of both elasticities would depend upon the magnitude of the sum of the intracity and intercity effects. For example, if the intercity effect is dominant, we would expect that the estimates of both elasticities of the demand for housing would be biased upward as a result of omitting the price variable from the estimation of the housing demand function. Since data on the price variable are not available in the Survey [42], we are unable to speculate about the domination of any effect, and hence the magnitude and the direction of the bias--a fact that should be recognized, although not avoidable, in acknowledging our results.

IV.4 The Estimation Technique: The Tobit Procedure

IV.4.1 The Simple Tobit Analysis

The housing demand function (91) can be estimated with the use of the simple tobit analysis where the error term, ξ , in the estimating equation is assumed to be homoscedastic. Consider the following tobit model where one observes M only if $M > 0$.

$$\begin{aligned} M_i &= I_i && \text{if } M_i > 0 \\ &= 0 && \text{otherwise} \end{aligned} \tag{111}$$

where

$$I_i = \beta_0 + \beta_1 Y_{li} + \beta_2 Y_{li}^2 + \beta_3 W_i + \beta_4 W_i^2 + \beta_5 Y_{li} W_i + \xi_i \quad (112)$$

Assume that the error term, ξ_i , is normally distributed with mean equals zero and variance σ^2 . The likelihood function of the joint distribution of the observations is

$$J = \prod_{i=1}^n \left(1 - F\left(\frac{E(I_i)}{\sigma}\right) \right) \prod_{i=n+1}^N \left(\frac{1}{\sigma} f\left(\frac{M_i - E(I_i)}{\sigma}\right) \right) \quad (113)$$

where $f(\cdot)$ is the standard normal density, $F(\cdot)$ is the cumulative normal density, and $E(I_i)$ is the expected value of the index I . The first term on the right-hand side of (113) represents the set of n observations for which $M_i = 0$; they are 34 percent of the total sample, N . The second term of the right-hand side represents the $(N - n)$ observations for which $M_i > 0$; they are 66 percent of the total. See Maddala [31, p. 163].

To obtain the regression coefficients, equation (113) has to be maximized with respect to the parameters β 's and σ by equating the derivatives to zero. Since the normal equations are nonlinear, one needs to use an iterative method to obtain β 's and σ . Using a program developed by Fische, Maddala, and Trost [13] one can obtain the converging values of β 's and σ . In fact, this program provides the values of β 's and σ that maximize the log of the likelihood function, J , as defined in (113). Besides, the values of

β 's are taken after both sides of (111) have been deflated by σ which can be a constant or any functional form.

After taking the values of β 's and σ , one can write

$$M_i = \hat{\beta}_0 + \hat{\beta}_1 Y_{li} + \hat{\beta}_2 Y_{li}^2 + \hat{\beta}_3 W_i + \hat{\beta}_4 W_i^2 + \hat{\beta}_5 Y_{li} W_i \quad (114)$$

where the "hat" above the coefficient refers to its estimated value from the maximization procedure of (113). Dividing (114) by $\hat{\sigma}$,

$$Z_i = \frac{\hat{I}_i}{\hat{\sigma}} = \frac{\hat{\beta}_0}{\hat{\sigma}} + \frac{\hat{\beta}_1}{\hat{\sigma}} Y_{li} + \frac{\hat{\beta}_2}{\hat{\sigma}} Y_{li}^2 + \frac{\hat{\beta}_3}{\hat{\sigma}} W_i + \frac{\hat{\beta}_4}{\hat{\sigma}} W_i^2 + \frac{\hat{\beta}_5}{\hat{\sigma}} Y_{li} W_i \quad (115)$$

where Z_i is a standard normal index that is to be used to calculate the expected value locus of M_i in (111) as

$$E(M_i) = M_i \cdot F(Z_i) + \hat{\sigma} \cdot f(Z_i) \quad (116)$$

The derivation of the expected value locus, $E(M_i)$, in (116) can be found in Tobin [53, p. 26], with the assumption that the lower limit value of the dependent variable, M , is zero. Therefore, equation (116) can be used to calculate the expected value of M at each level of Y_l and W , regarding the probability of buying a home, $F(Z)$, at each of these levels.

For the measurements of the elasticities of the demand for housing, the following expressions will be used.

$$\begin{aligned}\eta_{M,Y_\ell} &= \frac{\partial E(M_i)}{\partial Y_{\ell i}} \frac{Y_{\ell i}}{E(M_i)} \\ &= [\hat{\beta}_1 + 2\hat{\beta}_2 Y_{\ell i} + \hat{\beta}_5 W_i] [F(Z_i)] \frac{Y_{\ell i}}{E(M_i)}\end{aligned}\quad (117)$$

where η_{M,Y_ℓ} is the elasticity of the demand for housing with respect to labor income, and

$$\frac{\partial E(M_i)}{\partial Y_{\ell i}} = [\hat{\beta}_1 + 2\hat{\beta}_2 Y_{\ell i} + \hat{\beta}_5 W_i] [F(Z_i)] \quad (118)$$

The derivation of (118) can be found in Appendix 2, where we consider Amemiya's result [5] that the expected value of the error term, in the case of a normally-truncated dependent variable, is not zero. Similarly, for the wealth elasticity of the demand for housing,

$$\begin{aligned}\eta_{M,W} &= \frac{\partial E(M_i)}{\partial W_i} \frac{W_i}{E(M_i)} \\ &= [\hat{\beta}_3 + 2\hat{\beta}_4 W_i + \hat{\beta}_5 Y_{\ell i}] [F(Z_i)] \frac{W_i}{E(M_i)}\end{aligned}\quad (119)$$

These elasticities will be calculated at the median points of M , Y_ℓ , and W for the reason that our sample includes a large number of very rich households.

IV.4.2 The Heteroscedastic Tobit Analysis

Since the sample has a large number of the households which are very rich, there has to be considered the assumption that the variance of the error terms in the housing demand function (111) is homoscedastic. The reason is that the assumption of homoscedasticity in the estimating equation might not account for the differences in the variances of the error terms associated with the household demand for housing. That is, the error terms associated with the relatively wealthier and higher labor income households might have larger variances than those of the households with relatively lower wealth and labor income. Consequently, the estimates of (111) tend to be dominated by the large number of the households that are very rich. Hence, it would render the estimates of the housing demand function inconsistent and biased. Therefore, a test for homoscedasticity will be made in order to decide whether the variance of the error terms is or is not homoscedastic.

To make such a test, we start with the assumption that the variance of the error terms is heteroscedastic and takes on the following functional form

$$\sigma_i = \delta_0 + \delta_1 Y_{li} + \delta_2 W_i \quad (120)$$

where σ_i is the standard deviation of the error term, ξ_i , in (111), δ_0 is the intercept, δ_1 is the labor income coefficient, and δ_2 is the wealth coefficient. The test for homoscedasticity is a test for δ_1 and δ_2 being zero. Thus, we apply the likelihood-ratio test to examine the null hypothesis that δ_1 and δ_2 are zero versus the alternative hypothesis that δ_1 and δ_2 are different from zero. Such a test can be made by estimating, first, equation (113) where the maximum-log-likelihood value is taken with the restriction that δ_1 and δ_2 are zero, and, second, estimating equation (113) where the maximum-log-likelihood value is taken under the assumption we made in (120). Then the difference in the respective log-likelihood values, when multiplied by -2, is distributed as χ^2 with 2 degrees of freedom. Eventually, we would be able to make a statement about whether or not the variance of the error terms is constant for all observations in the sample. See Maddala [31, p. 43-44]. The rejection of the null hypothesis would imply that our estimates of the housing demand function with the use of the simple tobit analysis are inconsistent and biased. Therefore, if this is the case, we will use the heteroscedastic tobit analysis to estimate the housing demand function, with the use of trial values of σ_i , defined in (120), as the deflator of all variables in the housing demand function (111).

Finally, it is important to mention that the elasticities of the demand for housing measured by the heteroscedastic tobit analysis are the following. For the labor income elasticity,

$$\eta_{M,Y_i} = \frac{\partial E(M_i)}{\partial Y_{li}} \frac{Y_{li}}{E(M_i)}$$

where

$$\frac{\partial E(M_i)}{\partial Y_{li}} = [\hat{\beta}_1 + 2\hat{\beta}_2 Y_{li} + \hat{\beta}_5 W_i] [F(Z_i)] + \delta_1 [f(Z_i)] \quad (121)$$

Similarly, for the wealth elasticity,

$$\eta_{M,W} = \frac{\partial E(M_i)}{\partial W_i} \frac{W_i}{E(M_i)}$$

where

$$\frac{\partial E(M_i)}{\partial W_i} = [\hat{\beta}_3 + 2\hat{\beta}_4 W_i + \hat{\beta}_5 Y_{li}] [F(Z_i)] + \delta_2 [f(Z_i)] \quad (122)$$

The derivation of the terms (121) and (122) can be found in Appendix 2.

CHAPTER V

THE EMPIRICAL RESULTS

A principal finding of our model is that the use of labor income and wealth would improve and increase the estimate of the income elasticity of the demand for housing. The model avoids the simultaneous equations bias which lowers the estimate of the income elasticity found in the standard approach which uses total or permanent total income. In this chapter we make a comparison between our model and the standard approach regarding the estimation of the income elasticity. The first part of this chapter reports the results of our model. To provide an adequate comparison, we start with the estimation of the housing demand function using a heteroscedastic tobit analysis in order to show that such an estimation technique is appropriate in dealing with the problems of the truncation and heteroscedasticity. Next, we estimate the housing demand function in a linear and in a non-linear form to select the better fit for this function. We also include demographic variables as controls as suggested by previous studies on housing. Finally, we estimate the housing demand function with the use of average labor income and with the use of a predicted average labor income. Since both measures of labor income are proxies

for the permanent labor income concept, we will choose the one that produces better results. Later, we follow this procedure regarding the selection of the proxy for the permanent total income concept. The second part of the chapter provides the estimation of the housing demand function based on the standard approach. The results are produced with the use of our sample and estimation technique. We estimate the housing demand function, with and without wealth, to show the differences in the estimate of the income elasticity. Next, we consider the simultaneity problem and its effects on the estimate of the income elasticity. Finally, a concluding summary of the results of the comparison will be given.

It is important to mention that the estimates of the elasticities we report in this study are likely to be biased for the omission of the housing price index. The lack of data on price of housing in the sample is the reason for such an omission. In addition, the elasticities are measured with the formulas derived in the previous chapter at the median points of market value of home, wealth, and the income variables because a large number of the households in the sample are wealthy. For the demographic variables we use their means in the calculation of these elasticities.

The organization of this chapter is the following. Section V.1 reports the estimates of the housing demand function derived by our model. In subsection 1.A we provide

a test for homoscedasticity in order to rationalize the use of a heteroscedastic tobit analysis. Subsection 1.B presents the estimates of a linear form of the housing demand function. Subsection 1.C shows that a non-linear form of the housing demand function provides a better fit than the linear one. Finally, subsection 1.D presents the estimates of the housing demand function with the use of two proxies for the permanent labor income concept. In section V.2 we report the estimates of the demand for housing based on the standard approach with the use of our sample and estimation technique. Subsection 2.A examines the differences in the estimates of the income elasticity regarding wealth. In addition, subsection 2.B illustrates the simultaneous equations bias in the estimates of the income elasticity. Concurrently, we provide estimates of a two-stage procedure to avoid this bias.

V.1 Labor Income, Wealth, and the Demand for Housing

V.1.A. The Test for Homoscedasticity

An appropriate estimation technique is needed to deal with the problems of the truncation and heteroscedasticity found in our sample. First, consider ordinary least squares. The inclusion of non-home owners in the same manner as home owners might not qualify for the traditional assumptions of ordinary least squares, although this estimation technique

provides consistent, but inefficient, estimates if it were used. Next, the simple tobit analysis provides a treatment for the inclusion of non-home owners, but its estimates are inconsistent and biased in the presence of heteroscedasticity as was shown by Amemyia [5]. Thus, while ordinary least squares and the simple tobit analysis have some desirable properties, it seems likely our results will be influenced if either of the two approaches is used. Therefore, the use of the heteroscedastic tobit analysis, as introduced by Fische, Maddala, and Trost [13], seems to be the appropriate estimation technique because it provides consistent and unbiased estimates of the housing demand function using our sample.

We start with a test for homoscedasticity in order to investigate the nature of the variance of the error term in the estimation procedure. Table 3 shows the estimates of the housing demand function in two different forms: one with and the other without the assumption that the variance of the error term is heteroscedastic. A comparison¹ between equations (123) and (124) reveals that the log-likelihood

¹In fact, at different levels of the housing demand, the predicted probability of holding non-positive amounts of the housing stock is declining more rapidly, as labor income increases, in equation (123) than in equation (124). For the latter equation, the slow decline is closely related with the occurrence of the probability of holding non-positive amounts of the housing stock in the sample, which is 34%.

TABLE 3

THE TEST FOR HOMOSCEDASTICITY IN THE TOBIT ESTIMATES
OF THE HOUSING DEMAND FUNCTION
(The Dependent Variable is M)

	Equation* (123)	Equation (124)	Equation (125)	Equation (126)
intercept	5,966.2 (19.07)	5,119.3 (984.7)	1,279.2 (229.14)	2,686.6 (418.96)
$Y_{\ell 62}$	1.1890 (.0038)	.9880 (.0403)	.6950 (.0035)	.6673 (.0381)
W	-	-	.0244 (.0001)	.0861 (.0382)
σ^2				
Intercept	25,257 (80.38)	21,504 (106.61)	28,480 (126.61)	8,316 (232.34)
$Y_{\ell 62}$	-	.4099 (.0121)	-	.1796 (.0158)
W	-	-	-	.0958 (.0029)
		Continued		

TABLE 3

CONTINUED

	Equation (123)	Equation (124)	Equation (125)	Equation (126)
Log Likelihood	-17,555	-17,150	-17,150	-16,615
$\eta_{M,Y_{1962}}$	.337**	.361	.206	.421
$\eta_{M,W}$	-	-	.015	.146

*The numbers in parentheses are the standard errors of the coefficients.

**This elasticity is calculated with reference to the formulas in the previous chapter as the following:

$$M = 5,966.2 + 1.1890(7557) = 14,951.473$$

where 7557 is the median of labor income in 1962.

$$\hat{\sigma} = 25,257$$

$$z = \frac{14951.473}{25257} = .59$$

$$F(z) = .7224$$

$$f(z) = .3352$$

$$E(M) = 14951.473(.7224) + 25257(.3352) = 19267.09$$

$$\frac{\partial E(M)}{\partial Y_{1962}} = 1.1890(.7224) = .8589$$

Therefore,

$$\frac{\partial E(M)}{\partial Y_{1962}} \frac{Y_{1962}}{E(M)} = \frac{.8589(7557)}{19267.09} = .337$$

value increased from -17,555, equation (123), to -17,150, equation (124), as a result of adding labor income in the error-variance equation of (124). The difference in these two values, when multiplied by -2, is a statistic that is distributed as χ^2 with one degree of freedom. Since the value of the statistic is 810, the null hypothesis that the variance of the error term is unaffected by labor income is rejected at 99.5% confidence level.

Similar conclusions can be reached from the comparison between equations (125) and (126) where wealth is added as a regressor in the housing demand function and the error-variance equation. The likelihood-ratio test indicates that the null hypothesis that the variance of the error term is unaffected by labor income and wealth is rejected at 99.5% confidence level. In addition, a comparison between equations (124) and (126), which are estimated with and without wealth, shows that the log-likelihood value increased from -17,150, equation (124), to -16,615, equation (126). The difference in these two values, when multiplied by -2, is a statistic that is distributed as χ^2 with 2 degrees of freedom. Since the value of the statistic is 1070, one can reject the null hypothesis that neither the level of housing demand nor the variance of the error term is affected by wealth, at 99% confidence level.

The results of the test show that the variance of the error term is heteroscedastic and is influenced by labor

income and wealth. Therefore, we will utilize the heteroscedastic tobit analysis to estimate the housing demand function throughout this chapter.

V.1.B. The Linear Housing Demand Function

A considerable problem associated with the estimation of housing demand function is which functional form most appropriately represents the relationship between housing demand and its determinants. Though the theory does not give any indication for such a form, many authors used the linear one as a basic form in selecting the better fit of the relationship in question. We follow the same procedure by first estimating a linear functional form of the housing demand function in order to make the results comparable. Later, we try a non-linear form, and hence we decide about the most useful estimating equation.

Table 4 contains the estimates of the heteroscedastic tobit analysis of a linear housing demand function with the inclusion of demographic variables. The coefficient for labor income, as expected, is positive and significant. That is, those households which have higher labor income are likely to hold more of the housing stock in their portfolios; also the higher the labor income of the household, the higher the probability that a householder becomes a home owner, ceteris paribus. This is consistent with our assertion that increases in labor income would increase the marginal utility of additional housing services relative

TABLE 4
ESTIMATES OF A LINEAR HOUSING DEMAND FUNCTION
(The Dependent Variable is M)

	Regression Coefficient	Equation (127)	Standard Error of Coefficient
intercept	-22,238		3,719.4
$Y_{\ell 62}$	.5873		.0456
W	.0717		.0079
SEX	-1,537.9		1,584.9
MART	3,711.5		1,492.5
RACE	14.190		845.23
OCC	2,735.2		984.94
EDUC	301.72		89.896
AGE	676.53		134.23
(AGE) ²	-5.3093		1.3378
FSIZ	340.49		189.68
UAP ₁	-562.99		1,030.0
UAP ₂	1,599.0		922.82
UAP ₃	2,640.2		762.17
REG ₁	-251.00		868.84
REG ₂	852.16		905.02
REG ₃	-763.01		956.91
σ			
intercept	7,921.3		235.94
$Y_{\ell 62}$	.2313		.0204
W	.0806		.0021
Log Likelihood		-16,492	
$\eta_{M,Y_{\ell 62}}$		.354	
$\eta_{M,W}$		.114	

to that of consumption, and hence the household would be willing to hold more of the housing stock as the equilibrium condition (28) indicates.

The coefficient for wealth in equation (127) is positive and significant, indicating the role of wealth as the budget constraint on the household demand for the housing stock. As wealth increases, ceteris paribus, the household would be willing to increase its holdings of the housing stock. In addition, as wealth increases, the household would be able to pay for the down-payment requirements of a housing mortgage.

Table 4 also shows that the value of the income elasticity of the demand for housing, which is defined as the sum of the elasticities of the demand with respect to labor income and wealth, is .468. This value indicates that housing demand is inelastic with respect to total income. Such a result was similarly found in previous cross-sectional studies on housing. In addition, this value is lower than that reported in equation (126), at least by 10%. The explanation for this reduction is the inclusion of demographic variables in the estimates of equation (127), as was concluded by previous studies on housing.

The difference between the log-likelihood values in equations (126) and (127) shows the significance of the demographic variables in shaping the household demand for housing. Therefore, the null hypothesis that the

household demand for housing is unaffected by these demographic variables is rejected.

We include these variables as controls without clear theoretical predictions regarding likely importance or sign. An exception is the variable for the household size which has an obvious interpretation. As is shown, this coefficient is positive and has a high "t" ratio. Such a result is compatible with our assertion that a larger household unit should lead to a higher marginal rate of services provided by the housing stock, and hence a larger amount of the stock is demanded.

V.1.C. The Non-Linear Housing Demand Function

Conditions (69), (72), and (75) illustrate that the relationship between the demand for housing and its determinants is influenced by the levels of the variables and parameters of the model. In addition, the use of Taylor-series expansion to specify the functional form would give better fit for this relationship as more of non-linear terms are added. Therefore, to test for such a non-linearity we estimated the housing demand function as in Table 5.

Equation (128) was estimated with the inclusion of squared terms of labor income and wealth. A comparison between equations (126) and (128) indicates that the log-likelihood value has increased from -16,615, equation (126), to -16,554, equation (128), as a result of including these non-linear variables. The difference, when multiplied by

TABLE 5
ESTIMATES OF NON-LINEAR HOUSING DEMAND FUNCTIONS
(The Dependent Variable is M)

	Equation* (128)	Equation (129)	Equation (130)
Intercept	1,466.1 (514.01)	1,423.5 (516.09)	-18,433 (3,748.6)
$Y_{\ell 62}$	.8410 (.0602)	.8291 (.0611)	.6853 (.0786)
W	.1274 (.0086)	.1335 (.0091)	.1151 (.0099)
$Y_{\ell 62}^2$	$-.6019 \times 10^{-5}$ ($.9980 \times 10^{-6}$)	$-.5178 \times 10^{-5}$ ($.1195 \times 10^{-5}$)	$-.4347 \times 10^{-5}$ ($.1370 \times 10^{-5}$)
W^2	$-.3720 \times 10^{-7}$ ($.6092 \times 10^{-8}$)	$-.3512 \times 10^{-7}$ ($.7259 \times 10^{-8}$)	$-.3016 \times 10^{-7}$ ($.7781 \times 10^{-8}$)
$W \cdot Y_{\ell 62}$	-	$-.2925 \times 10^{-6}$ ($.2757 \times 10^{-6}$)	$-.1576 \times 10^{-6}$ ($.3008 \times 10^{-6}$)
SEX	-	-	-2,026.1 (1,574.3)
MART	-	-	3,474.8 (1,469.9)
RACE	-	-	-177.33 (822.23)
OCC	-	-	1,844.8 (960.74)
EDUC	-	-	199.30 (89.982)
AGE	-	-	552.30 (137.56)
(AGE) ²	-	-	-4.2351 (1.3874)

Continued

TABLE 5
CONTINUED

	Equation (128)	Equation (129)	Equation (130)
FSIZ	-	-	335.51 (184.80)
UAP ₁	-	-	-750.20 (999.62)
UAP ₂	-	-	1,455.2 (918.39)
UAP ₃	-	-	2,495.0 (749.54)
REG ₁	-	-	-496.53 (860.34)
REG ₂	-	-	711.98 (899.74)
REG ₃	-	-	-645.76 (956.53)
σ intercept	8,224.6 (234.75)	8,150.3 (235.3)	7,884.9 (245.15)
Y ₁₆₂	.1896 (.0142)	.1922 (.0221)	.2018 (.0220)
W	.0850 (.0024)	.0864 (.0025)	.0822 (.0023)
Log Likelihood	-16,554	-16,554	-16,454
$\eta_{M,Y_{162}}$	.465	.464	.382
$\eta_{M,W}$	.186	.191	.164

*The numbers in parentheses are the standard errors of the coefficients.

-2, is a statistic that is distributed as χ^2 with 2 degrees of freedom. Since the value of the statistic is 122, the null hypothesis that the estimates of the housing demand function are unaffected by non-linear terms is rejected. This result shows that the functional form of the housing demand equation is more likely to be non-linear where the elasticities depend on the level of each variable.

The coefficients for the squared terms of labor income and wealth, in equation (128), are negative and significant. In other words, at higher levels of labor income and wealth, the marginal propensities of holding the housing stock, with respect to labor income and wealth, are declining.

Equation (129) contains the estimates of the non-linear housing demand function with the inclusion of an interaction term between labor income and wealth. The coefficient for the interaction term is negative. This indicates that the higher the labor income, ceteris paribus, the lower the marginal propensity of holding the housing stock with respect to wealth, and vice-versa. A comparison between equations (128) and (129) reveals that the value of the log-likelihood does not change when the interaction term is included. Therefore, one cannot reject the null hypothesis that the demand for housing is unaffected by the interaction term.

Equation (130) exhibits our final estimation of the non-linear housing demand function with the inclusion of demographic variables. A comparison between equations (129)

and (130) reveals the significance of demographic variables in shaping the household demand for housing, by virtue of the likelihood-ratio test.

V.1.D. Housing Demand and Permanent Labor Income

We have argued that our model is an equilibrium one in the sense that a distinction between current and permanent labor income is not made, and there is no explicit utility maximization which determines the consumption-saving decision. Therefore, to examine the elasticity question properly we have used two proxy measures for the permanent labor income concept to estimate the housing demand function: (1) the average labor income, $\bar{Y}_\ell$, which is the average of the labor incomes of the households in the sample, in 1962 and 1963, and (2) the predicted average labor income, $\hat{Y}_\ell$, which is determined by wealth and the demographic characteristics of the household. In fact, these two proxies are similar to the most familiar measurements of the permanent total income concept in previous studies on housing. Their ultimate use is for the treatment of the error-in-measurement problem found in the empirical investigation of the relationship between income and the demand for housing when current income is used.

Both proxies of permanent labor income have merits in controlling for the downward bias in the estimate of labor income elasticity of the demand for housing. Nonetheless,

it seems likely that the predicted average labor income is the better one. The use of the average labor income decreases the transitory component of labor income through an averaging process in which one year's labor income is replaced by the average labor income. The process does not account for the differences in expected future earnings of labor income that are determined by wealth and the demographic characteristics of the household. Meanwhile, the use of the predicted average labor income considers the differences in expected future earnings of labor income, and leaves out some transitory components of labor income. Therefore, the use of the predicted average labor income would control better for the bias than average labor income. That is, the labor income elasticity of the demand for housing based on the predicted average labor income is expected to be higher than that based on average labor income.

Table 6 shows the estimates of the housing demand function with the use of average labor income of 1962 and 1963. The results show that the values of the labor income elasticity are .496, equation (131), and .367, equation (132). These values are high when compared with those of labor income in 1962, equations (126) and (127), respectively. The reason is that the use of average labor income decreases the transitory component of labor income, and hence controlled for the downward bias in the estimate of the true

TABLE 6
ESTIMATES OF THE HOUSING DEMAND FUNCTION
WITH THE USE OF AVERAGE LABOR INCOME
(The Dependent Variable is M)

	Equation* (131)	Equation (132)
intercept	1,722.4 (487.54)	-24,641 (3,701.1)
$\bar{Y}_l$	.7448 (.0439)	.5270 (.0541)
W	.0725 (.0076)	.0680 (.0078)
SEX	-	-1,383.0 (1,588.0)
MART	-	3,967.0 (1,499.4)
RACE	-	30.409 (829.93)
OCC	-	2,725.3 (978.29)
EDUC	-	298.71 (90.822)
AGE	-	756.21 (135.09)
(AGE) ²	-	-6.3023 (1.3568)
FSIZ	-	323.90 (188.00)

Continued

TABLE 6
CONTINUED

	Equation (131)	Equation (132)
UAP ₁	-	-421.49 (1,033.5)
UAP ₂	-	1,729.4 (907.88)
UAP ₃	-	2,689.8 (761.93)
REG ₁	-	-144.70 (866.61)
REG ₂	-	807.80 (902.73)
REG ₃	-	-573.49 (936.48)
σ intercept	8,364.3 (274.26)	7,381.9 (327.96)
$\bar{Y}_\ell$	.1864 (.0169)	.2756 (.0283)
W	.0843 (.0024)	.0767 (.0020)
Log Likelihood	-16,617	-16,491
$\eta_{M, \bar{Y}_\ell}$	.496	.367
$\eta_{M, W}$	.128	.111

*The numbers in parentheses are the standard errors of coefficients.

relationship between labor income and the demand for housing.

For the estimate of labor income elasticity of the demand for housing based on the predicted average labor income, we used a two-stage procedure to estimate the housing demand function. In the first stage, equation (133), we estimated a predicting equation of average labor income. We regressed the average labor income, as the dependent variable, on wealth and the demographic characteristics of the household. The basic assumption underlying the predicting equation is that cross-sectional differences in the household permanent labor income are attributed to differences in wealth and the personal characteristics of the household. In addition, we utilized the heteroscedastic tobit analysis to estimate the predicting equation in order to treat the problem of heteroscedasticity. In the second stage, equation (134), we inserted the predicting equation as a regressor, $\hat{Y}_l$, in the housing demand function and the error-variance equation. We did not include any of the demographic variables in the estimates of the housing demand function, equation (134), in order to make it identifiable.

The results confirm our assertion that the predicted average labor income works better than average labor income in controlling for the downward bias. A comparison between equations (134) and (132) shows that the value of the elasticity based on the predicted average labor income is higher than that based on average labor income.

TABLE 7
ESTIMATES OF THE PREDICTED AVERAGE
LABOR INCOME
(The Dependent Variable $\bar{Y}_l = (Y_{l62} + Y_{l63})/2$)

	Regression Coefficient	Equation (133) Standard Error of Coefficient
intercept	-7,581.3	954.68
W	.0826	.0074
SEX	717.31	450.38
MART	2,037.5	403.03
RACE	570.71	242.91
OCC	1,809.2	327.83
EDUC	397.27	28.047
AGE	205.05	37.525
(AGE) ²	-1.6860	.3772
FSIZ	196.59	51.948
UAP ₁	1,610.3	303.47
UAP ₂	834.73	297.13
UAP ₃	1,345.3	234.22
REG ₁	592.12	283.23
REG ₂	351.85	294.09
REG ₃	-354.68	295.92
σ		
intercept	2,500.1	62.049
W	.1132	.0020
Log Likelihood	-20,337	

TABLE 8
TWO-STAGE ESTIMATES OF THE HOUSING DEMAND
FUNCTION WITH THE USE OF THE PREDICTED
AVERAGE LABOR INCOME

(The Dependent Variable is M)

	Regression Coefficient	Equation (134)	Standard Error of Coefficient
intercept	-1,741.4		970.62
$\hat{Y}_\ell$	.8661		.1349
W	.0476		.0147
σ			
intercept	6,397.5		523.9
$\hat{Y}_\ell$	.4601		.0758
W	.0667		.0078
Log Likelihood		-16,590	
$\eta_{M, \hat{Y}_\ell}$		.858	
$\eta_{M, W}$		.082	

It is worth mentioning that the estimates of the elasticities are likely to be biased for the omission of the housing price index. The reason for this omission is the lack of data on price of housing in our sample.

V.2. Total Income and the Demand for Housing: A Comparison

In the previous part of the chapter we provided estimates of the income elasticity of the demand for housing based on our model's prediction that labor income and wealth are the determinants of the demand for housing. In this part we wish to examine our results versus those based on the standard approach of using total or permanent total income in order to show the differences between the two approaches. Our comparison, in the previous chapter, between the two approaches indicates there are two estimation problems associated with the standard approach: (1) the omission of a relevant regressor, wealth, that is likely to be correlated with an included regressor, total income, hence the income elasticity is biased, and (2) the problem of simultaneity which biases the income elasticity downward. We will show these results as if there were a model which justified the determination of the household demand for housing by wealth and total income. In addition, a somewhat similar two-stage procedure, as we used in the first part of this chapter, provides the estimates of the income elasticity with respect to a permanent total income concept and

with respect to a current total income concept that we utilize to control for the simultaneous equations bias.

V.2.A. Does Wealth Matter?

To answer this question we investigate the role of wealth in determining the household demand for housing. The estimation of the housing demand function by our model shows that the coefficients for labor income and wealth, as expected, are significant. The model rationalizes the use of labor income on the grounds that a distinction should be made between labor income, as exogenously determined, and total income which is determined by the household asset holdings. Thus, the use of labor income and wealth would avoid the simultaneity problem associated with the use of total income. Nonetheless, one might argue that the coefficient for wealth is ultimately significant because labor income is used and wealth is highly correlated with the excluded non-labor income. Therefore, to test for this argument, we estimate the housing demand function with the use of total income and wealth.

Table 9 contains the estimates of the housing demand function based on total income in two different forms: one with and the other without wealth. A comparison between equations (135) and (136) reveals that the income elasticity of the demand for housing has declined from .506, equation (135), to .332, equation (136), as a result of including wealth in the estimates of the housing demand function.

TABLE 9
ESTIMATES OF THE HOUSING DEMAND FUNCTION WITH THE
USE OF TOTAL INCOME

(The Dependent Variable is M)

	Equation* (135)	Equation (136)
intercept	2,226.8 (516.72)	2,634.8 (541.83)
Y_{62}	.6327 (.0452)	.3625 (.0576)
W	-	.0730 (.0107)
σ intercept	8,326.0 (222.70)	8,008.8 (259.93)
Y_{62}	.2959 (.0233)	.2383 (.0281)
W	.0690 (.0017)	.0898 (.0034)
Log Likelihood	-16,626	-16,600
$\eta_{M,Y_{62}}$	.506	.332
$\eta_{M,W}$	-	.149

*The numbers in parentheses are the standard errors of the coefficients.

Such a decline is explained by our assertion that the inclusion of wealth would reduce the income elasticity of the demand for housing, in the sense that total income and wealth are positively correlated. For the statistical significance of wealth in the housing demand function, the log-likelihood value has increased from -16,626 in equation (135) to -16,600 in equation (136). The difference in these two values, when multiplied by -2, is a statistic that is distributed as χ^2 with one degree of freedom. Since the value of the statistic is 52, the null hypothesis that the demand for housing is unaffected by wealth is rejected. Therefore, wealth does matter. The explanation for this answer is straight forward: wealth is the financial constraint on the household demand for housing. As wealth increases, ceteris paribus, the householder would be willing to become a home owner, and also would be able to pay for the down-payment requirements of a mortgage.

Similar conclusions for the significant role of wealth can be reached by the comparison of equations (137) and (138), Table 10, where demographic variables were added to both equations.

For the measurement of the elasticity of the demand for housing based on permanent total income, Table 11 shows our estimates of the housing demand function using the average total income of 1962 and 1963, $\bar{Y}$. Equations (139) and (140) depict higher elasticity than those reported with the use of

TABLE 10
 ESTIMATES OF THE HOUSING DEMAND FUNCTION WITH
 THE USE OF TOTAL INCOME: A COMPARISON
 FOR THE OMISSION OF WEALTH
 (The Dependent Variable is M)

	Equation* (137)	Equation (138)
intercept	-23,894 (3,718.9)	-24,334 (3,837.1)
Y ₆₂	.4413 (.0481)	.6700 (.0450)
W	.0606 (.0085)	-
SEX	-1,434.8 (1,598.2)	-1,640.6 (1,625.7)
MART	3,895.8 (1,505.3)	3,686.0 (1,542.1)
RACE	86.174 (856.94)	51.748 (894.43)
OCC	3,206.4 (961.35)	4,417.7 (949.97)
EDUC	326.34 (92.414)	294.83 (95.953)
AGE	731.73 (134.87)	742.16 (136.89)
(AGE) ²	-5.9613 (1.3481)	-5.7874 (1.3552)
FSIZ	343.70 (192.49)	258.20 (203.67)

Continued

TABLE 10
CONTINUED

	Equation (137)	Equation (138)
UAP ₁	-420.13 (1,047.3)	-1,006.7 (1,073.1)
UAP ₂	1,671.2 (934.25)	1,485.8 (954.05)
UAP ₃	2,764.6 (770.53)	2,412.8 (794.41)
REG ₁	-70.909 (882.35)	-211.88 (918.02)
REG ₂	940.94 (918.08)	925.73 (950.82)
REG ₃	-658.20 (965.95)	-799.44 (1,000.9)
σ Intercept	7,944.4 (243.74)	8,000.5 (230.72)
Y ₆₂	.2283 (.0209)	.2891 (.0198)
W	.0744 (.0022)	.0680 (.0016)
Log Likelihood	-16,508	-16,518
$\eta_{M,Y_{62}}$	.321	.457
$\eta_{M,W}$	.098	-

*The numbers in parentheses are the standard errors of the coefficients.

TABLE 11
ESTIMATES OF THE HOUSING DEMAND FUNCTION WITH THE
USE OF AVERAGE TOTAL INCOME: A COMPARISON FOR
THE OMISSION OF WEALTH
(The Dependent Variable is M)

	Equation* (139)	Equation (140)
intercept	-23,284 (3,699.4)	-23,542 (3,799.7)
$\bar{Y}$	.4907 (.0538)	.7148 (.0510)
W	.0543 (.0082)	-
SEX	-1,534.0 (1,534.1)	-1,746.6 (1,548.8)
MART	3,849.8 (1,471.0)	3,634.0 (1,498.1)
RACE	92.482 (836.07)	65.56 (860.51)
OCC	3,053.6 (954.76)	4,065.1 (939.54)
EDUC	292.65 (93.472)	254.74 (96.744)
AGE	710.89 (132.37)	714.09 (133.64)
(AGE) ²	-5.7116 (1.3111)	-5.4837 (1.3091)
FSIZ	308.54 (192.48)	219.27 (201.22)

Continued

TABLE 11
CONTINUED

	Equation (139)	Equation (140)
UAP ₁	-401.92 (1,032.4)	-940.91 (1,051.5)
UAP ₂	1,736.0 (913.48)	1,557.7 (920.79)
UAP ₃	2,704.2 (764.19)	2,353.7 (784.88)
REG ₁	-66.913 (869.61)	-191.04 (894.46)
REG ₂	935.60 (900.69)	925.35 (923.51)
REG ₃	-607.55 (937.29)	-715.48 (960.84)
σ intercept	7,560.9 (321.41)	7,510.7 (312.66)
$\bar{Y}$	.2675 (.0264)	.3286 (.0252)
W	.0682 (.0018)	.0631 (.0014)
Log Likelihood	-16,504	-16,512
$\eta_{M, \bar{Y}}$	.372	.498
$\eta_{M, W}$	.062	-

*The numbers in parentheses are the standard errors of the coefficients.

total income in 1962, equations (137) and (138). This can be explained by the problem of error-in-measurement that lowers the elasticity when one year's income is used. In addition, the likelihood-ratio test between equations (139) and (140) indicates that the null hypothesis that the demand for housing is not influenced by wealth is rejected. Thus, even with the use of average total income, wealth is significant for the determination of the household demand for housing.

Finally, Tables 12 and 13 provide our two-stage estimates of the housing demand function. In the first stage, Table 12, we estimated the predicted average total income as determined by average labor income, wealth, and the demographic characteristics of the household. In the second stage, we inserted equation (141) as a regressor, $\hat{Y}$, in the housing demand function (142) and in its error-variance equation. We found that the elasticity of the demand for housing based on the predicted average total income is higher than that based on the average total income. This supports our assertion that the predicted average income is a better proxy for the permanent income concept than the average income. In addition, the coefficient for wealth has a high "t" ratio.

A word must be said about our estimates of the income elasticity of the demand for housing compared with previous studies of housing. We found that the estimates we reported

TABLE 12
ESTIMATES OF THE PREDICTED
AVERAGE TOTAL INCOME
(The Dependent Variable $\bar{Y} = (Y_{62} + Y_{63})/2$)

	Regression Coefficient	Equation (141) Standard Error of Coefficient
intercept	-2,896.6	773.24
$\bar{Y}_L$	.7868	.0213
W	.0429	.0037
SEX	138.04	255.02
MART	422.70	251.67
RACE	-340.01	133.84
OCC	53.987	392.60
EDUC	116.70	17.082
AGE	99.880	25.348
(AGE) ²	-1.1895	.2242
FSIZ	204.87	33.963
UAP ₁	145.58	235.72
UAP ₂	310.95	206.93
UAP ₃	301.27	176.23
REG ₁	-7.1396	215.95
REG ₂	-202.06	206.73
REG ₃	-310.38	197.89
σ		
intercept	943.76	25.898
$\bar{Y}_L$	.1369	.0044
W	.0354	.0011
Log Likelihood	-18,848	

TABLE 13
TWO-STAGE ESTIMATES OF THE HOUSING DEMAND
FUNCTION WITH THE USE OF PREDICTED
AVERAGE TOTAL INCOME
(The Dependent Variable is M)

	Regression Coefficient	Equation (142)	Standard Error of Coefficient
intercept	1,425.2		548.29
$\hat{Y}$	.5875		.0619
W	.0567		.0086
σ			
intercept	7,258.3		338.42
$\hat{Y}$	.3351		.0366
W	.0625		.0027
Log Likelihood		-16,570	
$\eta_{M,\hat{Y}}$		.556	
$\eta_{M,W}$		.101	

in this part of the chapter are very similar, indeed, to those estimates reported by cross-sectional studies. For example, see Carliner [8], Trost [54], Rosen [47], Lee [29], and Polinsky and Ellwood [39]. The difference between our results and theirs lies in the fact that our estimates omit the housing price index, while theirs omit wealth from the housing demand function. In addition, there is a problem in the use of the market value of the home as we mentioned earlier (see page 67). Thus, had the information on the price of housing been available in our sample, one would have been able to provide a better comparison. Nonetheless, the conclusion that the demand for housing is influenced by the household wealth, and hence the exclusion of wealth would lead to inappropriate estimates of the income elasticity of the demand for housing, is worth considering.

V.2.B. Housing Demand and the Simultaneous Equations Bias

According to the argument we made in the previous chapter, the total income elasticity of the demand for housing would be biased downward owing to the simultaneous equations bias. We have attempted a two-stage procedure to estimate the housing demand function with the control for the correlation between the total income variable and the error term in such a function. In the first stage, Table 14, we regressed the total income in 1962 on wealth, labor income, and the demographic variables. Equation (143) shows that total income, as predicted, is positively related to

TABLE 14
ESTIMATES OF THE PREDICTED TOTAL
INCOME IN 1962

(The Dependent Variable is Y_{62})

	Regression Coefficient	Equation (143) Standard Error of Coefficient
intercept	803.23	470.17
Y_{62}	.9704	.0068
W	.0366	.0069
SEX	-30.213	238.76
MART	169.17	223.05
RACE	-202.27	106.70
OCC	44.090	319.91
EDUC	3.5284	12.889
AGE	-35.048	18.183
(AGE) ²	.5698	.1728
FSIZ	8.2637	24.502
UAP ₁	153.58	148.23
UAP ₂	-121.79	165.59
UAP ₃	-5.3066	121.85
REG ₁	46.120	146.65
REG ₂	-32.469	162.44
REG ₃	68.465	157.57
σ		
intercept	1,042.7	15.895
Y_{62}	-.0542	.00086
W	.0902	.0009
Log Likelihood		-18,336

wealth and labor income. For the treatment of the heteroscedasticity problem in this equation, we have used the heteroscedastic tobit model with the assumption that the variance of the error term is a function of labor income and wealth. In the second stage, equation (144) in Table 15, we inserted the predicting equation of the total income in 1962, $\hat{Y}_{62}$, as a regressor into the housing demand function and its error-variance equation.

The result is that the total income elasticity of the demand for housing becomes larger than those in equations (136) and (137) at least by 20%. This result, as predicted, indicates that the estimates of such an elasticity, without accounting for the simultaneous equations bias, would be inconsistent and biased downward.

On the other hand, with the exclusion of wealth in the second stage, we have repeated the same procedure in equation (145). Similarly, we found that the total income elasticity of the demand for housing becomes larger, compared with equation (135) or with equation (138).

Finally, it is worth mentioning that we have not included any of the demographic variables in the second stage in order to make such an equation identifiable. In addition, a comparison between equations (142) and (144) suggests that the two income elasticities of these equations are almost the same. This might be attributed to the fact that both definitions of income can be used as proxy measures for the permanent income concept.

TABLE 15
TWO-STAGE ESTIMATES OF THE HOUSING DEMAND
FUNCTION WITH THE USE OF THE PREDICTED
TOTAL INCOME

(The Dependent Variable is M)

	Equation* (144)	Equation (145)
intercept	2,502.7 (430.39)	1,993.1 (443.97)
$\hat{Y}_{62}$	.7062 (.0380)	.8722 (.0359)
W	.0432 (.0076)	-
σ		
intercept	8,151.5 (215.3)	8,213.9 (212.35)
$\hat{Y}_{62}$	.2563 (.0183)	.3031 (.0181)
W	.0650 (.0018)	.0635 (.0017)
Log Likelihood	-16,615	-16,619
$\eta_{M, \hat{Y}_{62}}$	.542	.641
$\eta_{M, W}$	.075	-

*The numbers in parentheses are the standard errors of the coefficients.

In conclusion, we were able to estimate the income elasticity of the demand for housing, from our sample data, according to the standard approach of using total income. The heteroscedastic tobit analysis was the estimation technique we used to treat the heteroscedastic nature of the variance of the error term in the estimation procedure. The results showed that the estimation of the income elasticity of the demand for housing, based on the standard approach, has two estimation problems. First, the omission of wealth, which is positively correlated with total income, is likely to bias the income elasticity upward. The magnitude of the bias is about 13%, as Table 10 shows. However, the elasticity with respect to "scale" is essentially unchanged by the addition of wealth because the added elasticity with respect to wealth is about 10%. Second, since total income is an endogenous variable, a simultaneous equations bias causes the income elasticity to be underestimated. The magnitude of the underestimation is about 20%, comparing Table 10 with Table 15.

We attempted a comparison between the estimate of the income elasticity of the demand for housing based on our model and that based on the standard approach. The comparison reveals that the estimates are different, although our method of calculating this elasticity is similar to that of the standard approach, as we showed at the end of the third chapter. For example, our model's results show that the income elasticity of the demand for housing, Table 8, is

.94, while the same elasticity based on the standard approach is .641, Table 15. Thus, the standard approach lowers the magnitude of such an elasticity, by 30%, because of the simultaneity problem. Therefore, the comparison supports our finding that the use of labor income and wealth, rather than total income, would improve and increase the estimate of the income elasticity of the demand for housing.

CHAPTER VI

CONCLUSIONS

We have derived a model that considers the portfolio-balance behavior of the household in determining its demand for the housing stock. This model is built on the idea that the housing choice is jointly determined by the consumption and investment decisions of the household. The household is assumed to maximize the utility derived from the consumption of the housing services and the consumption of all other goods and services, including the services of the other assets, subject to the wealth constraint. Such an approach is needed to construct an appropriate estimate of the income elasticity of the demand for housing based on cross-sectional data.

With some reasonable assumptions about marginal utility of consumption, interest rate on housing mortgage, and the effect of the U.S. income tax, we were able to derive the housing demand function. Accordingly, the demand for housing is determined by labor income, wealth, and the price of housing. For the measurement of the income elasticity of the demand for housing, our model suggests that it is the sum of the elasticities of the demand with respect to labor income and wealth. Such a method is not theoretically

in conflict with that of the standard approach of using total or permanent total income, but it suggests a more useful estimation technique.

We have utilized the tobit analysis to estimate the elasticities of the demand for housing from a cross-sectional sample that consists of 66 percent home owners and 34 percent non-home owners. Without any separation of the data, we were able to measure these elasticities regarding the probability of holding the housing stock in the portfolio structure of the household. In addition, we have tested for homoscedasticity because a large number of the households in the sample are wealthy. The test showed that homoscedasticity has a great effect on our result: it renders the estimates inconsistent and biased. Therefore, we have utilized a heteroscedastic tobit analysis to provide consistent and unbiased estimates.

Our results are sensitive to the problem of omitting the housing price index from the estimates of the housing demand function. The problem, as in most cross-sectional studies, stems from the lack of data on the housing price index in the sample. However, the structure of a housing price index from other sources of data might not be relevant to our sample. This limited our attempt to deflate the dependent variable, the market value of the home, to get a proxy for the quantity demanded for the housing stock. Therefore, we expect that our estimates of the elasticities of the demand for housing are biased. The magnitude and

direction of the bias depend on the sum of the housing price variations among the cities and regions of the U.S.A. wherein our sample was drawn. Such an explanation is based on the assertion that the relationship between labor income, wealth, and the price of housing is a spatial phenomenon.

To test whether the estimation of the income elasticity of the demand for housing as suggested by our approach leads to different results from those suggested by the standard approach, we have attempted a comparison between the two approaches. The comparison showed that the estimate of the income elasticity of the demand for housing based on our approach is higher than that based on the standard approach by at least 30 percent, Tables 8 and 15. Such a difference is attributed to the simultaneous equations bias and its mechanism of underestimating the income elasticity of the demand as suggested by the standard approach. We feel that the income elasticity of the demand as suggested by this approach is an improvement on the standard approach.

Finally, we hope that our model finds applications in other fields of household investment such as the demand for automobiles, or other assets having similar characteristics.

APPENDIX 1
DISCUSSION OF THE DATA

The sample was constructed from two related surveys. The first is the Survey of Financial Characteristics of Consumers [42] which has information on the determinants of size of wealth, the components of wealth and debts and their diffusion throughout the population, and the variation in patterns of ownership among 2,557 consumer units of differing characteristics. Families with large amounts of wealth were surveyed in order to provide averages more reliable than those usually obtained for the top wealth groups. In addition, assets were valued, for most components of wealth, at market value. Data on wealth, income, and demographic variables in 1962 were obtained from this survey to estimate the housing demand function.

The second survey is the Survey of Changes in Family Finances [41] which was designed to reinterview these consumer units for their savings and incomes in 1963. Data on incomes of 1963 were used to construct averages for labor income, total income, and predicted averages of both incomes to estimate the housing demand function. Excluded were families with negative incomes in 1962 and 1963, negative amount of wealth or its components, and families

that did not respond in the second survey. Therefore, the sample was reduced to 1,984.

The data are subject to some limitations, as in most reinterview surveys, arising from the facts that "they were obtained from a sample rather than from the total population, from the refusal of some who were drawn in the sample to take part in the survey, and from errors of response" [42, p. 2]. In addition, the data lack information on a housing price index. This led to the omission of such an index from the estimation of the housing demand function in the sense that the use of other sources of data to construct a housing price index might not be relevant to the data in our sample. Thus, we expect our results to be biased due to the omission of the housing price index. Furthermore, the data lack information on the household income tax in 1962 and 1963. These circumstances limited our attempt to correct for imputed rental income in the measurement of the income variables in our study.

Table 16 contains a statistical summary of the mean, median, standard deviation, maximum, and minimum values of the variables used to estimate the housing demand function. In the measurement of the elasticities of housing demand, we used the median values of all variables, except the demographic ones, in an attempt to correct for the overrepresentation of the wealthy households in the sample. For the demographic variables we used their means.

TABLE 16

STATISTICAL SUMMARY OF THE VARIABLES USED TO ESTIMATE
THE HOUSING DEMAND FUNCTION
(N = 1,984)

	Mean	Median	Standard Deviation	Maximum	Minimum
Market value of home (M)	20,528.3	13,000	30,725.3	350,000	0
Net worth of the household (W)	165,013	15,865	534,713	6,709,000	1
Labor income in 1962 (Y_{1962})	13,671.3	7,557	20,899.2	246,800	0
Labor income in 1963 (Y_{1963})	15,774.5	10,002	22,120.2	270,077	13
Average labor income ($\bar{Y}_l$)	14,722.9	7,873.25	20,846.6	228,925	6.5
Predicted average labor income ($\hat{Y}_l$)	14,722.9	11,736.6	12,474.4	103,856	-9,751.88
Total income in 1962 (Y_{62})	18,735.4	8,563.5	31,838.9	401,540	0
Predicted total income in 1962 ($\hat{Y}_{62}$)	18,735.4	9,492.42	28,822.9	294,437	-5,684.98

Continued

TABLE 16
CONTINUED

	Mean	Median	Standard Deviation	Maximum	Minimum
Total income in 1963 (Y_{63})	19,074.4	9,202.5	30,718.4	309,475	0
Average total income ($\bar{Y}$)	18,904.9	8,858.25	30,512.4	315,417	0
Predicted average total income ($\hat{Y}$)	18,904.9	9,880.82	28,140.2	284,793	-5,339.68
$Y^2_{\ell 62}$	6.235×10^8	5.711×10^7	2.772×10^9	6.091×10^{10}	0
W^2	3.130×10^{11}	2.517×10^8	2.079×10^{12}	4.50×10^{13}	1
$W \cdot Y_{\ell 62}$	7.057×10^9	7.687×10^7	3.825×10^{10}	8.428×10^{11}	0
Sex of the household head; dummy for male (SEX)	.873	1	.3331	1	0
Marital status of head; dummy for married (MARR)	.815	1	.3884	1	0
Race of head; dummy for white (RACE)	.838	1	.3684	1	0

Continued

TABLE 16
CONTINUED

	Mean	Median	Standard Deviation	Maximum	Minimum
Occupation of head; dummy for self-employed (OCC)	.185	0	.3888	1	0
Education of head; number of years (EDUC)	11.972	12	4.007	20	0
Age of head; number of years (AGE)	49.535	49	14.380	91	17
Age-squared (AGE) ²	2,660.44	2,401	1,469.74	8,281	289
Family size; number of persons in the house- hold (FSIZ)	3.376	3	1.8157	18	1
Dummy for urban areas with population of one million or more in the USA (UAP ₁)	.113	0	.3172	1	0
Dummy for urban areas with population between 250,000 and one million in the USA (UAP ₂)	.158	0	.3646	1	0

Continued

TABLE 16
CONTINUED

	Mean	Median	Standard Deviation	Maximum	Minimum
Dummy for urban areas with population less than 250,000 in the USA (UAP ₃)	.355	0	.4786	1	0
Dummy for the North-eastern region of the USA (REG ₁)	.288	0	.4529	1	0
Dummy for the North-central region of the USA (REG ₂)	.289	0	.4536	1	0
Dummy for the Southern region of the USA (REG ₃)	.242	0	.4284	1	0

Notice: for M, W, and the demographic variables of the household, data are for 1962, as collected in the Survey [42]. For the income averages, the variables are constructed from the same Survey and the saving survey [41] of 1963 as the subscripts indicate.

APPENDIX 2

MEASUREMENTS OF ELASTICITIES IN TOBIT ANALYSIS

Consider the linear version of the tobit model in (111); the estimated housing demand equation is

$$M = \hat{\beta}_0 + \hat{\beta}_1 Y_L + \hat{\beta}_2 W \quad (2.1)$$

The expected value locus of (2.1), as shown by Tobin [53, p. 26], under the assumption that the lower limit value of M is zero, can be written as

$$E(M) = M \cdot F(Z) + \sigma \cdot f(Z) \quad (2.2)$$

where the expected value of the error term is assumed to be $\sigma(f(Z)/F(Z))$ in the case of normally-truncated dependent variable, as shown by Amemyia [5, p. 1001]. In addition, as defined before

$$F(Z) = \int_{-\infty}^Z \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}Z^2\right] dZ \quad (2.3)$$

and

$$f(Z) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}Z^2\right] \quad (2.4)$$

where

$$Z = \frac{M}{\sigma} \quad (2.5)$$

2.A The Simple Tobit Analysis

For the simple tobit analysis, the variance of the error term is assumed to be constant, i.e. homoscedastic. Thus, the standard deviation of the error term, σ , is constant and can be written as

$$\sigma = \hat{\sigma} \quad (2.6)$$

Now, to derive the responsive changes in $E(M)$ as Y_ℓ changes, we differentiate (2.2) with respect to Y_ℓ , that is

$$\begin{aligned} \frac{\partial E(M)}{\partial Y_\ell} &= M \left(\frac{\partial F(Z)}{\partial Z} \frac{\partial Z}{\partial M} \frac{\partial M}{\partial Y_\ell} \right) + \frac{\partial M}{\partial Y_\ell} [F(Z)] \\ &+ \sigma \left(\frac{\partial f(Z)}{\partial Z} \frac{\partial Z}{\partial M} \frac{\partial M}{\partial Y_\ell} \right) \end{aligned} \quad (2.7)$$

Since

$$\frac{\partial F(Z)}{\partial Z} = f(Z) \quad (2.8)$$

$$\begin{aligned} \frac{\partial f(Z)}{\partial Z} &= \frac{-Z}{\sigma \sqrt{2\pi}} e^{-\frac{1}{2}Z^2} \\ &= -Z f(Z) \end{aligned} \quad (2.9)$$

$$\frac{\partial Z}{\partial M} = \frac{1}{\sigma} \quad (2.10)$$

and

$$\frac{\partial Z}{\partial \sigma} = \frac{-M}{\sigma^2} \quad (2.11)$$

Substituting (2.5), (2.6), and (2.8)-(2.10) into (2.7)

$$\begin{aligned} \frac{\partial E(M)}{\partial Y_\ell} &= M[f(Z) \left(\frac{1}{\hat{\sigma}}\right) (\hat{\beta}_1)] + \hat{\beta}_1 [F(Z)] \\ &\quad + \hat{\sigma} \left[\frac{-M}{\hat{\sigma}} f(Z) \left(\frac{1}{\hat{\sigma}}\right) (\hat{\beta}_1)\right] \end{aligned} \quad (2.12)$$

Therefore

$$\frac{\partial E(M)}{\partial Y_\ell} = \hat{\beta}_1 [F(Z)] \quad (2.13)$$

hence

$$\eta_{M, Y_\ell} = \hat{\beta}_1 [F(Z)] \left[\frac{Y_\ell}{E(M)}\right] \quad (2.14)$$

Similarly, for changes in W

$$\frac{\partial E(M)}{\partial W} = \hat{\beta}_2 [F(Z)] \quad (2.15)$$

hence

$$\eta_{M, W} = \hat{\beta}_2 [F(Z)] \left[\frac{W}{E(M)}\right] \quad (2.16)$$

2.B The Heteroscedastic Tobit Analysis

The variance of the error term in (2.2) has been changed in this analysis in order to consider heteroscedasticity. Thus

$$E(M) = M \cdot F(Z) + \sigma_i \cdot f(Z) \quad (2.2')$$

where

$$\sigma_i = \hat{\sigma}_i = \delta_0 + \delta_1 Y_\ell + \delta_2 W \quad (2.17)$$

Now, to derive the responsive changes in $E(M_i)$ as Y_ℓ changes, we differentiate (2.2') with respect to Y_ℓ , regarding the assumption about σ_i in (2.17)

$$\begin{aligned} \frac{\partial E(M)}{\partial Y_\ell} &= M \left[\frac{\partial F(Z)}{\partial Z} \frac{\partial Z}{\partial M} \frac{\partial M}{\partial Y_\ell} + \frac{\partial F(Z)}{\partial Z} \frac{\partial Z}{\partial \sigma_i} \frac{\partial \sigma_i}{\partial Y_\ell} \right] \\ &+ \frac{\partial M}{\partial Y_\ell} [F(Z)] + \sigma_i \left[\frac{\partial f(Z)}{\partial Z} \frac{\partial Z}{\partial M} \frac{\partial M}{\partial Y_\ell} \right. \\ &\left. + \frac{\partial f(Z)}{\partial Z} \frac{\partial Z}{\partial \sigma_i} \frac{\partial \sigma_i}{\partial Y_\ell} \right] + \frac{\partial \sigma_i}{\partial Y_\ell} [f(Z)] \end{aligned} \quad (2.18)$$

Since

$$\frac{\partial \sigma_i}{\partial Y_\ell} = \delta_1 \quad (2.19)$$

$$Z = \frac{M}{\sigma_i} \quad (2.5')$$

$$\frac{\partial Z}{\partial M} = \frac{1}{\sigma_i} \quad (2.10')$$

and

$$\frac{\partial Z}{\partial \sigma_i} = \frac{-M}{\sigma_i^2} \quad (2.11')$$

Substituting to calculate the term in (2.18)

$$\begin{aligned} \frac{\partial E(M)}{\partial Y_\ell} = & M[f(Z) \left(\frac{1}{\hat{\sigma}_i}\right) (\hat{\beta}_1) + f(Z) \left(\frac{-M}{\sigma_i^2}\right) (\hat{\delta}_1)] \\ & + \hat{\beta}_1 \cdot F(Z) + \hat{\sigma}_i \left[\frac{-M}{\hat{\sigma}_i} (f(Z)) \left(\frac{1}{\hat{\sigma}_i}\right) (\hat{\beta}_1) \right. \\ & \left. + \left(\frac{M}{\hat{\sigma}_i}\right) (f(Z)) \left(\frac{M}{\hat{\sigma}_i^2}\right) (\hat{\delta}_1) \right] + \hat{\delta}_1 [f(Z)] \end{aligned} \quad (2.20)$$

therefore

$$\frac{\partial E(M)}{\partial Y_\ell} = \hat{\beta}_1 [F(Z)] + \hat{\delta}_1 [f(Z)] \quad (2.21)$$

hence

$$\eta_{M, Y_\ell} = [\hat{\beta}_1 \cdot F(Z) + \hat{\delta}_1 \cdot f(Z)] \frac{Y_\ell}{E(M)} \quad (2.22)$$

Similarly, for changes in W

$$\frac{\partial E(M)}{\partial y_l} = \hat{\beta}_2 [F(Z)] + \hat{\delta}_1 [f(Z)] \quad (2.23)$$

hence

$$\eta_{M,W} = [\hat{\beta}_2 \cdot F(Z) + \hat{\delta}_2 \cdot f(Z)] \frac{W}{E(M)} \quad (2.24)$$

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BIOGRAPHICAL SKETCH

Farouk Saleh Khatib was born on February 17, 1948, in the holy city of Mecca, Saudi Arabia. He received his bachelor's degree in economics from King Abdul Aziz University, Jeddah, Saudi Arabia, in May 1972. Between May 1972 and June 1973, he worked as a teaching assistant on the Faculty of Economics and Business Administration at the same university. In August 1974, Mr. Khatib joined the Department of Economics, University of Illinois, Urbana-Champaign, to work for the Master of Science degree in economics which he obtained in May 1976. In March 1977, Mr. Khatib enrolled as a graduate student at the Department of Economics, University of Florida, Gainesville, to pursue his studies for the doctoral degree in economics.

Mr. Khatib has gained many teaching experiences in the schools of Saudi Arabia, King Abdul Aziz University, and the University of Florida, Gainesville.

Mr. Khatib also has worked for the government of Saudi Arabia in different fields: Ministry of Pilgrimage and Endowments, Ministry of Labor and Social Affairs, and Center for Research and Industrial Development, Jeddah. In the private sector, he worked for the Saudi National Commercial Bank, and the Saudi Arabian Airlines, Jeddah.

Mr. Khatib was awarded the First Honor Degree of King Abdul Aziz University for the accomplishment of his bachelor's degree in economics in 1972.

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I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.



William A. Bomberger, Chairman
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Ann R. Horowitz
Associate Professor of Economics

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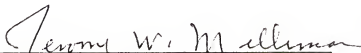
John Kraft
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and Real Estate

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.



Madelyn M. Lockhart
Professor of Economics

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.



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This dissertation was submitted to the Graduate Faculty of the Department of Economics in the College of Business Administration and to the Graduate Council, and was accepted as partial fulfillment of the requirements for the degree of Doctor of Philosophy.

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